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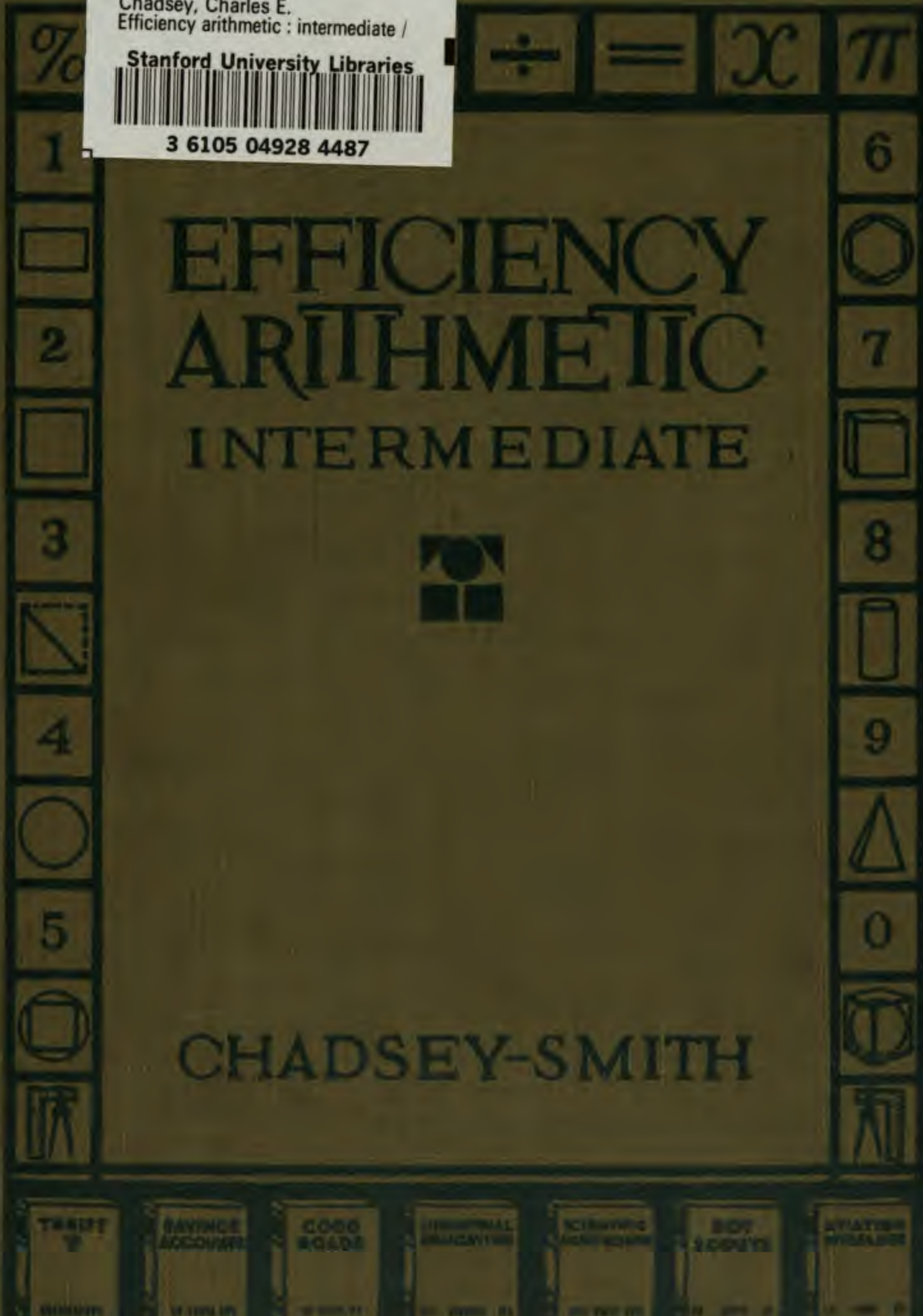

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EFFICIENCY ARITHMETIC

INTERMEDIATE

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PREFACE

An arithmetic designed for pupils of the Fifth and Sixth Grades must combine drill upon fundamental operations, including decimals and common fractions, with as wide an application of these principles to the common experiences of life as possible. While the children in these grades are immature and have had very limited practical experiences, it is necessary to include simple examples drawn from real facts and from genuine business transactions. Wherever possible the applied problems are taken from the actual experiences of children themselves and in all cases the facts upon which the problems are based are accurate and of real informational value.

The fact that many pupils leave school at the end of the Sixth Grade to go to work, and that there is an increasing tendency to substitute junior high school courses, make the intermediate arithmetic a fundamentally important text book. If for an appreciable number of pupils the intermediate arithmetic offers the last formal training in mathematics, it is necessary so far as possible to include the arithmetical processes and topics used under ordinary conditions of life.

Attention is called to the psychological plan of this book, by which the previous knowledge of the child is utilized in introducing a new subject.

The development of the subject of decimals is based upon the knowledge of common fractions already secured. In this way teachers find little difficulty in developing a subject which not infrequently has caused mental confusion.

The fact that in the presentation of the subject of denominate numbers, the use of both decimals and common fractions in the solution of problems is emphasized, simplifies another subject often troublesome to children.

In a similar way the treatment of percentage is based upon the knowledge of decimals and common fractions and, while giving a practical elementary knowledge, is in no way beyond the understanding of the child.

In this book, as in the other books of the series, the idea of systematic, standardized drill is emphasized. The practice exercises in fractions presented in Chapter I, Part II, will prove of great value in developing a clear conception of simple fractions and ability to perform the fundamental operations in fractions with speed and accuracy.

In the preparation of this Arithmetic, the authors have had the efficient aid of Miss Myra Banks, of the Winnetka (Ill.) Public Schools, and of Miss Katherine L. McLaughlin, of the School of Education, University of Chicago; and they are also indebted to Miss Annie J. Robinson, Principal of Case-Woodland School, Cleveland, for valuable criticisms and suggestions.

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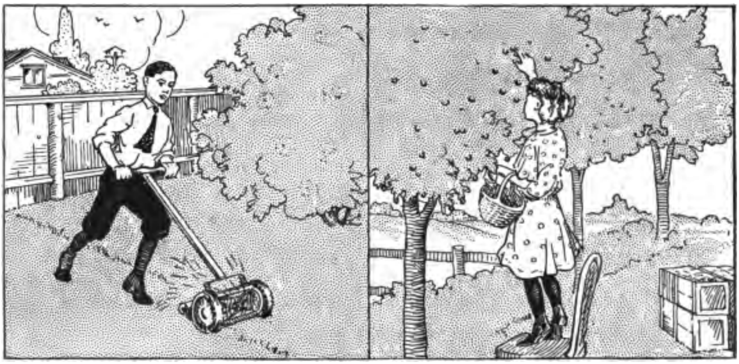
INTERMEDIATE ARITHMETIC

PART 1

CHAPTER I



Earning Money

Jane and Robert agreed to save at least half of what they earned during the summer vacation. They kept an account of what they earned in order to know how much they should put in the bank.

Exercise 1

1. During the 12 weeks of summer vacation, Jane earned 25 cents a week for drying the lunch dishes each day. How much did she earn from this source during vacation?

2. What did she earn at 10 cents a week for keeping the pansy bed weeded and the pansies picked?

3. She picked 16 quarts of cherries at 3 cents a quart from the trees in the back yard. What did she earn in this way?

4. She picked 24 quarts of strawberries at 2 cents and 6 quarts of red raspberries at 4 cents a quart. How much did she earn picking berries?

5. She hemmed a dozen tea towels at 5 cents apiece. How much did she earn by her sewing?

6. She was paid 10 cents a week for dusting the living room each day and keeping a fresh bouquet on the dining-room table. How much did she earn in this way?

7. Find Jane's earnings for the 12 weeks.

8. Of this, she put $5.00 in her bank. How much had she left to spend?

9. During these 12 weeks, Robert earned 25 cents a week for cutting the grass and 75 cents for digging dandelions from it. How much did he earn in all taking care of the lawn?

10. He picked 20 quarts of cherries at 3 cents a quart, 25 quarts of strawberries at 2 cents and 10 quarts of red raspberries at 4 cents. How much did he earn in all picking fruit?

11. He received 10 cents a week for washing the porches and sidewalks with the hose and earned $1.00 altogether for going on long errands. How much did he earn in these two ways?

12. He picked 12 quarts of peas and 8 quarts of beans at 2 cents a quart and 2 bushels of cucumbers at 10 cents a bushel. How much did he earn picking these vegetables?

13. Find Robert's total earnings for the 12 weeks.

14. He put $5.00 in the bank. How much had he left to spend?

15. When school began, Robert bought 2 pencils, an eraser, a spelling blank, a penholder and a ruler, each of which cost 5 cents. How much of his money had he left?

16. Jane bought 2 five-cent pencils, a five-cent spelling blank, a box of crayons for 20 cents and 2 five-cent tablets. How much did she spend on these school supplies? How much of her money did she have left?

NOTATION AND NUMERATION

Exercise 2. Reading and Writing Numbers

You have already had some practice in reading and writing numbers. In your studies this year you will find many large numbers and you should be able to read them rapidly and accurately. In order to do this you will need to understand our system of writing and reading numbers.

1. In the number 21 what does the figure 1 stand for? The figure 2?

2. Tell what each figure stands for in the number 536.

Do you know how many people in the United States have savings accounts? In 1915 there were 11,811,169 people who had money on deposit in savings banks.

Notice that a large number such as 11,811,169 is separated into groups for convenience in reading. What names are given to these groups?

$$\overbrace{11}^{\text{millions}},\overbrace{811}^{\text{thousands}},\overbrace{169}^{\text{units}}$$

The number, then, is read eleven million, eight hundred eleven thousand, one hundred sixty-nine.

In pointing off whole numbers we begin at the right hand figure. How many figures are put into each group? These groups are generally called periods.[1]

3. In the number 3,006 why is it necessary to fill in the vacant places with zeros? What would the number be without the two zeros between the 3 and the 6?

---

[1] If a number contains more than three periods, the fourth period is called *billions*.

Exercise 3

Point off and read the following numbers:

1. 1729
2. 2348
3. 3024
4. 15729
5. 16305
6. 65280
7. 45728
8. 344865
9. 216507
10. 834009
11. 21406520
12. 42000375
13. 167245000
14. 382450086
15. 753482167

16. Read the populations for 1916 of these ten largest cities in the United States: New York 5,468,190; Chicago 2,447,045; Philadelphia 1,683,664; St. Louis 745,983; Boston 745,139; Cleveland 656,975; Baltimore 584,604; Pittsburgh 571,984; Detroit 554,717; Los Angeles 465,367.

Exercise 4

Write the following numbers:

1. One thousand, nine hundred fifteen.
2. Five thousand, four hundred eight.
3. Ten thousand, six hundred seventy-three.
4. Fifty-six thousand, one hundred fifty-three.
5. One hundred three thousand, eight hundred seventy.
6. One million, eighty thousand, six hundred seventy-five.
7. Twenty-one million, four hundred thousand, forty-eight.
8. Two hundred thirty-seven thousand, five hundred sixty.
9. One hundred million, one hundred thousand, one hundred.
10. Two million, eight hundred thirty-four thousand, nine.
11. Write a number containing at least two periods and read it to some pupil to write on the blackboard.

The teacher may also dictate in words the numbers in Exercise 3 for additional practice in writing numbers.

Exercise 5. Roman Numerals

The Romans expressed their numbers with letters instead of figures. We still use Roman numerals for certain purposes. Name several ways in which we now use Roman numerals.

In the Roman system of writing numbers the value of each of the seven capital letters that are used is as follows:

I = 1; V = 5; X = 10; L = 50; C = 100; D = 500; M = 1000.

These letters are grouped according to certain principles:

a. Repeating a letter repeats its value.

For example: III = 3; XXX = 30; CCC = 300.

b. If a letter is written before another of greater value, its value is subtracted from the value of the greater letter.

For example: IV = 4; IX = 9; XC = 90.

c. If a letter is written after another of greater value, its value is added to the value of the greater letter.

For example: VI = 6; XVI = 16; CX = 110.

d. Placing a bar over a letter multiplies its value a thousand times.

For example, $\overline{\text{X}}$ = 10,000.

Exercise 6

Read the following:

1. II.
2. VII.
3. IX.
4. XLV.
5. XL.
6. XLIV.
7. XC.
8. XCIX.
9. LV.
10. DCC.
11. XXXII.
12. LXXVII.
13. MD.
14. MDCCCC.
15. MCDXCII.
16. MDCCCCXVII.

Write the following numbers in Roman numerals:

1. 4.
2. 8.
3. 12.
4. 14.
5. 16.
6. 19.
7. 21.
8. 34.
9. 48.
10. 63.
11. 1812.
12. 2000.

REVIEW OF FUNDAMENTALS

Exercise 7. Addition

Practice every day until you can say the following sums in 40 seconds or write them in 1 minute:[1]

1.	2	9	4	6	4	7	5	8	5
	2	9	3	5	2	6	5	7	4
2.	9	3	5	8	5	4	8	3	8
	3	2	3	4	2	4	5	3	8
3.	6	7	6	7	9	6	9	9	6
	2	4	4	7	2	3	5	8	6
4.	9	8	8	7	8	9	7	9	7
	7	2	6	5	3	6	3	4	2

Exercise 8. Adding by Endings

A	B
64	89
97	51
10	87
83	68
59	73
76	92
42	25
98	30
35	14
21	46

Cut a cardboard slip $\frac{3}{4}$ inch x 5 inches. Letter one of the slips A and the other B. Write on them the numbers shown in the picture. Keep the slips in your arithmetic.

1. Place card A at the edge of your paper. In a column beside it write the sum of each number and 4, as 64 and 4; 97 and 4, etc.

2. Use card B in the same way another day.

3. Then try adding 3, 5, 6, 7, 8 and 9 to each of the numbers on these cards.

---

[1] The order should be frequently changed in drilling. Means of doing this are: Flash cards; rapid dictation of the combinations by the teacher, and copied or mimeographed sheets on which the order is changed.

Exercise 9. Problems in Addition

1. A squad of Boy Scouts went on a hike. In the morning they walked 9 miles and in the afternoon 7 miles. How many miles did they walk during the hike?

2. During vacation Alexander raised 12 bushels of onions on one piece of land and 29 bushels on another. What was his total yield of onions?

3. Martha bought a hat for $3.50 and a pair of shoes for $3.75. What did she spend for both?

4. While shopping for his mother, Albert bought a dozen oranges for 39 cents, a head of cauliflower for 15 cents and a loaf of bread for 10 cents. Find the amount of his purchases.

5. Lucile worked for a neighbor, receiving 35 cents for working in the garden and 25 cents for mowing the lawn. How much did she receive for both?

6. Walter and Clifton went fishing. Walter caught 19 fish and Clifton 27. How many did they both catch?

7. Marion entered a fly-swatting contest. Here is her record for one week: Monday 34, Tuesday 63, Wednesday 56, Thursday 40, Friday 49, and Saturday 87. Find her total for the week.

8. Hilda raises chickens to earn her spending money. She keeps them separated in three pens, containing 19, 23 and 16 each. How many has she altogether?

9. In a city school there are 4 fifth grades containing 33, 32, 36 and 29 pupils respectively. How many fifth-grade pupils are there in that school?

Exercise 10. Practice Exercises in Addition

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
3	8	7	2	5	8	2	4
4	1	6	4	8	5	2	7
5	9	0	3	2	4	5	5
7	6	8	5	6	6	8	6
2	4	6	6	9	2	8	8

9.	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**
4	6	1	2	7	5	4	6
9	7	4	6	7	7	4	6
7	9	5	8	9	5	6	8
5	5	8	3	3	3	3	5
3	1	4	1	4	1	3	2
6	8	3	7	9	8	2	6
5	3	7	9	4	2	5	7

17.	**18.**	**19.**	**20.**	**21.**	**22.**
53	38	98	27	15	30
25	39	13	59	55	18
34	62	86	73	38	27
76	85	17	45	89	36
27	41	48	80	96	85

23.	**24.**	**25.**	**26.**	**27.**	**28.**
55	44	27	98	40	96
72	68	20	65	64	83
87	99	65	29	17	18
36	24	28	14	29	20
42	10	39	71	73	24

29.	**30.**	**31.**	**32.**	**33.**	**34.**
88	98	36	16	54	82
27	46	79	85	48	73
54	39	98	28	11	49
60	86	22	41	19	36
19	54	50	35	82	15
44	77	25	69	46	75
63	31	45	87	38	48

Exercise 11. Explanation for Addition

Some pupils learn to add examples of two or more columns correctly, but can not explain the process. If you have not already learned why you take the various steps, study the following explanation carefully:

Add: 456, 372, 865 and 974.

456
372
865
974
——
2667

Explanation: Write the numbers so that the units all fall in one column, the tens in a column, and so on. Begin adding at the units' column. The sum of the units 4, 5, 2 and 6 is 17. 17 makes 7 units and 1 ten. Write the 7 units in the units' place in the sum and add the 1 ten with the other tens in the second column. 1, 7, 6, 7 and 5 tens make 26 tens. Since 10 tens make a hundred, 26 tens make 2 hundreds, with a remainder of 6 tens. Write the 6 representing the remaining tens in the tens' place of the sum and add the 2 hundreds in with the figures in the hundreds' column. The sum of 2, 9, 8, 3 and 4 hundreds is 26 hundreds. Therefore the sum of the numbers is 2667.

Add the following examples and explain each:

1.	2.	3.	4.	5.	6.	7.
605	238	941	733	847	306	855
542	638	426	394	128	574	637
365	372	375	866	963	849	516
738	490	759	665	382	783	837
352	638	364	987	379	461	178

8.	9.	10.	11.	12.	13.	14.
745	462	359	638	281	368	927
682	719	426	805	537	495	673
445	684	913	748	437	638	250
604	812	361	947	236	855	738
999	864	525	608	749	664	927

Call the pupil's attention to short cuts by using grouping in addition. Let the pupil use the groups which come most easily to him, but do not force your system of grouping on him. Encourage grouping when pupils are adding orally.

Exercise 12. Practice Exercise in Addition

The examples in this exercise may be used for seat work or dictated by the teacher for blackboard work. Add first by columns and then horizontally by lines:

	1.	**2.**	**3.**	**4.**	**5.**
6.	1748	1825	3793	2406	9612
7.	9778	8797	9059	8929	9767
8.	1662	8729	5569	7884	1453
9.	4860	7638	8677	1698	3626

	10.	**11.**	**12.**	**13.**	**14.**
15.	1583	2670	4429	6832	5874
16.	4627	4494	3746	4459	6019
17.	9213	5531	2513	7410	3163
18.	4560	8324	8054	2635	2725

	19.	**20.**	**21.**	**22.**	**23.**
24.	8194	4715	3806	8910	1482
25.	3410	2308	4297	4213	5529
26.	6825	4387	1342	4526	3043
27.	6372	7692	7985	2869	1677
28.	5907	6501	5406	7570	6585

	29.	**30.**	**31.**	**32.**
33.	59318	60842	37925	67884
34.	55857	95725	59489	92887
35.	49964	56459	60878	23515
36.	89137	48717	68472	74663
37.	60898	46727	62653	74016
38.	12895	69937	65744	69182

By adding the totals for examples 33-38 and comparing the result with the sum of the totals for examples 29-32 a cross check can be obtained on the work. The other groups may be treated in a similar way.

Exercise 13. Subtraction

Practice until you can say all the remainders in the following simple subtraction examples in 60 seconds or write them in 90 seconds:

8	3	14	6	12	7	6	13	6	9
5	2	9	2	6	4	5	6	3	5
7	15	8	7	10	9	9	15	8	10
1	9	4	6	6	6	1	8	6	5
10	16	14	16	11	12	17	10	18	4
8	9	5	8	2	8	9	7	9	2
9	14	11	14	12	13	11	6	12	13
7	8	7	7	9	8	6	1	7	4
11	13	16	17	15	13	12	11	13	14
8	9	7	8	7	7	5	4	5	6

The cards which were used for addition on page 7 can also be used for subtraction by endings.

Exercise 14

1. Practice subtracting 2, 3, 4, 5, 6, 7, 8 and 9 from each of the numbers on cards A and B.

Exchange your papers and correct them as the teacher reads the correct answers. Keep a record of these daily drills so that you may note your improvement.

It has been found a good plan to try out pupils individually on these combinations while the others are doing written work. If a pupil hesitates too long or gives a wrong answer, supply the fact and count one error each time.

Exercise 15

Subtract:

1.	2.	3.	4.	5.	6.	7.
987	476	888	999	738	596	864
562	341	537	462	506	334	512

8.	9.	10.	11.	12.	13.	14.
817	995	847	777	587	666	899
605	762	523	446	262	231	563

Exercise 16. Explanation for Subtraction

Subtract 5283 from 9437.

9437 minuend
5283 subtrahend
4154 remainder

Explanation: 3 units from 7 units leaves 4 units. We can not take 8 tens from 3 tens, so we take 1 hundred from the 4 hundreds and change it into tens, making 10 tens. 10 tens and 3 tens make 13 tens. We can now take 8 tens from the 13 tens, leaving 5 tens. 2 hundreds from the 3 hundreds that are left leaves 1 hundred. 5 thousands from 9 thousands leaves 4 thousands.

It is more difficult to give an explanation for an example with several zeros in the minuend:

Example: Subtract 238 from 500.

500 minuend
238 subtrahend
262 remainder

Explanation: We can not take 8 units from 0 units. We have no tens to change into units so we first take 1 of the 5 hundreds and change it into 10 tens. We can now take 1 of these 10 tens and change it into 10 units. 8 units from 10 units leaves 2 units. 3 tens from the 9 tens that are left leaves 6 tens. 2 hundreds from the 4 hundreds that are left leaves 2 hundreds. The remainder then is 262.

Bundles of splints may be used to explain the process of subtraction. Toothpicks bound with rubber bands also provide a good substitute for splints when the splints are not available.

Subtract the following, explaining[1] each step:

1.	**2.**	**3.**	**4.**	**5.**
7642	9826	9837	6891	8729
2971	6128	7429	3963	5642
6.	**7.**	**8.**	**9.**	**10.**
8350	7630	8510	9950	6320
3287	2514	3486	7265	1899
11.	**12.**	**13.**	**14.**	**15.**
5400	8500	9000	6700	1000
2563	4187	7482	1237	725
16.	**17.**	**18.**	**19.**	**20.**
2000	5000	9000	7000	8000
1937	2506	7218	3470	5847

In order that we may have common names to use in speaking of any number used in a subtraction, mathematicians have agreed to call the number subtracted, the *subtrahend*; the number from which another number is subtracted, the *minuend*, and the result that is left after subtracting, the *remainder*. Name the minuends and subtrahends in the above examples.

Exercise 17. Checking Subtraction

Since the remainder is the difference between the minuend and the subtrahend, the sum of the subtrahend and the remainder equals the minuend. We may then check our results in subtraction by adding the subtrahend and the remainder and comparing this sum with the minuend.

Subtract and check:

21.	**22.**	**23.**	**24.**	**25.**
4856	6854	9870	5037	7315
2178	1936	6483	2618	4382

---

[1] Teachers using the Austrian method in subtraction should substitute the explanation for the Austrian method in place of the explanation given on the preceding page.

Exercise 18. Problems in Subtraction

1. Kenneth counted the marbles in his marble bag. He had 237. He gave his cousin 65 of them for helping him with some work. How many had he left?

2. Robert weighs 68 pounds and Jane weighs 59 pounds. How much more does Robert weigh than Jane?

3. Walter took 75 cents to the butcher shop to buy meat. He bought 53 cents worth of sirloin steak. How much money did he have left?

4. Edwin has 169 stamps in his collection and Philip has 153. How many more stamps does Edwin have than Philip?

5. Arthur raised 38 bushels of popcorn. He sold 13 bushels to a grocer. After keeping 1 bushel for his own use he shipped the remainder to the city. How many bushels did he ship to the city?

6. Jane was given $2.25 by her mother for buying refreshments for a party. She spent only $1.79. How much did she have left to return to her mother?

7. Helen and Alexander were to choose sides for a spelling match. To decide which was to have first choice, they were to guess the number on the page in a book where a marker had been placed. Helen guessed 125 and Alexander guessed 165. The page was 139. Which was the better guess?

8. A fifth-grade class gave an entertainment. They received $28.75 for admissions, and their expenses were $12.29. What were their profits on the entertainment?

9. In a recent year there were 5,468,190 inhabitants in the city of New York and 2,447,045 in Chicago. How much did the population of New York exceed that of Chicago?

Exercise 19. Subtraction

1.	2.	3.	4.	5.	6.	7.
879	978	798	789	899	698	876
564	653	364	289	217	487	693

8.	9.	10.	11.	12.	13.	14.
400	500	300	200	700	800	900
127	356	238	144	517	362	876

Exercise 20. Addition

1.	2.	3.	4.	5.
619	346	947	395	241
244	512	305	480	677
475	884	954	734	526
733	687	536	609	168
585	923	938	542	594

6.	7.	8.	9.	10.
714	743	562	443	365
679	992	404	802	560
280	287	353	952	854
792	346	892	546	776
196	957	671	587	299

Exercise 21. Subtraction Drill

Subtract each of the following from 96,473:

1. 92,432
2. 24,292
3. 28,351
4. 33,824
5. 79,722
6. 69,292
7. 81,355
8. 49,449
9. 55,582
10. 43,996
11. 67,547
12. 26,893
13. 47,865
14. 45,982
15. 68,968
16. 79,657
17. 57,809
18. 60,887
19. 89,770
20. 68,095

Exercise 22. Problems in Addition and Subtraction

1. If you have 95 cents and buy a book for 50 cents, how much will you have left?

2. John bought 2 pounds of steak for 54 cents and a pound of butter for 35 cents. How much did he spend for both butter and steak?

3. Herbert had 321 marbles and sold 125 of them. How many did he have left? The next day he bought 35 more. How many did he then have?

4. A farmer values his farm at $7500, his livestock at $875, his furniture at $350 and his farming implements at $425. Find the total estimated value of his property.

5. An orchard contains 480 trees. Of these trees, 124 are Willow Twigs, 153 are Wine Saps and the remainder are Jonathans. How many Jonathan trees are there in the orchard?

6. A freight train consisted of 16 cars of coal, 17 cars of wheat and a caboose car. How many cars were there in the train?

7. A conductor on a street car has rung up 67 fares. There are only 43 people remaining on the car. How many have left the car since the trip began?

8. A dealer bought 4 cars of coal containing 30 tons, 29 tons, 28 tons and 30 tons. How many tons were there in all?

9. The population of London in 1916 was estimated to be 7,251,358 and that of New York 5,468,190. How much greater was the population of London than New York?

10. In 1916 the farm animals in the United States were estimated to number as follows: Horses 21,160,000; mules 4,565,000; milch cows 21,988,000; other cattle 39,453,000; sheep 49,162,000 and swine 68,047,000. Find the total number of farm animals as estimated for that year.

11. The ten companies of a certain regiment contain respectively 95, 89, 88, 93, 87, 78, 85, 92, 99 and 90 soldiers. How many soldiers are there in the whole regiment?

12. A room is 12 feet wide and 15 feet long. How many feet of picture molding will it take to go around the room, making no allowance for the doors and windows?

13. If there are two windows each 4 feet wide and a double door 5 feet wide, how many feet would be deducted from the molding to allow for these openings? How many feet would be left?

14. The following is a record of money earned outside of school hours by the pupils of a certain city school during one year. Find the sum of each column.

Occupations	Boys	Girls	Total
Farm and Garden	$1,506.38	$ 32.92	$1,539.30
Store	1,128.46	38.45	1,166.91
Newspapers	786.27	0.00	786.27
Golf, caddying	285.96	0.00	285.96
Peddling	283.42	0.00	283.42
Chores and Housework	289.87	262.81	552.68
Errands	220.11	123.79	343.90
Janitor Work	174.06	0.00	174.06
Moving Pictures	168.40	0.00	168.40
Miscellaneous	432.26	57.20	480.46

15. The following is a summary of what the pupils did with the money received. Find the total and see if it checks with the total column in problem 14.

Deposited in Bank	$2,015.10
Deposited in Home Savings Banks	408.83
Given to Parents	2,278.88
Clothing	602.26
Pleasure	356.47
Outings	15.00
Machines	39.58
Miscellaneous	137.49

Multiplication

An accurate knowledge of the facts of the multiplication table is absolutely essential to accurate and rapid work in multiplication. The drills given below will help you to fix the multiplication facts more firmly. Practice on them until you can say the products for each drill in 60 seconds or write them in 90 seconds.

Exercise 23. Drill 1

2	9	3	8	7	3	6	0	4	7	4
2	5	4	3	2	5	4	3	2	5	4
2	5	8	9	7	11	5	1	6	9	12
3	2	5	4	3	2	5	4	3	2	5
8	3	10	6	11	9	3	4	10	12	0
4	3	2	5	4	3	2	5	4	3	2
11	9	7	10	12	0	12	11	1	12	10
5	1	4	3	2	5	4	3	8	4	5

Exercise 24. Drill 2

7	9	3	6	5	4	8	3	11	4
8	6	9	7	8	6	9	7	8	6
5	4	8	7	2	7	12	6	9	5
9	7	8	6	9	7	8	6	9	7
11	5	6	2	6	8	4	12	10	3
8	6	9	7	8	6	9	7	8	6
7	8	2	12	11	9	3	7	12	9
9	7	8	6	9	7	8	6	9	7

The order of these exercises should be frequently changed when they are hectographed or written on the blackboard so that the answers will not be memorized.

Another interesting way for practicing on the multiplication facts is to make a cardboard slip similar to those in addition and subtraction:

C

6
2
0
5
1
12
7
11
4
9
8
3
10

Cut a cardboard slip 5 inches by $\frac{3}{4}$ inch. On it write out of order the numbers 1 to 12, including 0.

Label this slip C, and keep it with A and B in your arithmetic.

I. Lay the slip beside your paper. Multiply each number by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12, as your teacher may tell you, writing the product after each number.

II. After multiplying add 2, 3, 4, 5, 6, 7 or 8 to each product before writing the result.

Multiplication Game

III. Copy this column twice on the board.

Choose two teams, four on a team if you are taking four "tables," five if you are taking five, etc.

The teams race in pairs. The first two, for example, start writing at the signal, and multiply each number by 4. When one finishes, the other must stop. Each problem correctly worked counts 1 point for the side to which the contestant belongs.

The next two write the fives, the next the sixes, etc.

The side with the greatest total of points wins.

Multiplication Race

Another exciting contest in the multiplication facts is to write a row similar to the one shown below across the blackboard:

5	9	3	8	2	7	6	4	7	4	3	6	9
7	6	4	9	6	3	6	5	9	8	8	7	9

Start a pupil at each end of this row and see which one can write the most correct products by the time they come together.

Explanation for Multiplication

Multiply 475 by 9.

475 multiplicand
9 multiplier
4275 product

Explanation: Multiplication is merely a short method for addition We know that we should have 9×5 units or 45 units in the sum of the units if we were adding. We write the 5 in the units place in the product and save the four tens to add to the tens. 9×7 tens=63 tens. 63 tens+4 tens=67 tens. 67 tens make 6 hundreds, with a remainder of 7 tens. Write the 7 in the product and save the 6 hundreds. 9×4 hundreds=36 hundreds. 36 hundreds+6 hundreds=42 hundreds. Therefore the product is 4275.

Exercise 25

Use this exercise for drill in multiplication:

1. 3,672
2. 4,895
3. 2,506
4. 8,179
5. 3,457
6. 8,692
7. 3,714
8. 5,206
9. 7,628
10. 49,53
11. 6,125
12. 8,309
13. 4,763
14. 2,918
15. 5,046
16. 9,934
17. 6,388
18. 2,766
19. 5,177
20. 2,855

Multiply each number in the above list by 2, by 3, by 4, by 5, by 6, by 7, by 8, by 9.

Multiply 358 by 42.

358 multiplicand
42 multiplier
2×358=716 1st partial product
4 tens ×358=1432 2nd partial product
15036 product

Explanation: We first multiply by the 2 (the units' figure of the multiplier) as in example one. The 4 in the multiplier represents 4 tens. 358×4 tens gives 1432 tens. The 2 of this partial product must then be written in tens' place, the 3 in hundreds' place, etc. If the multiplier contained a figure in hundreds' place, where would the first figure of the third partial product be written?

The teacher should insist on enough explanations from the pupils to see that they understand the process and then drill for speed.

Exercise 26. Multiplication

1.	**2.**	**3.**	**4.**	**5.**
6582	3479	4872	9563	2945
4	9	6	3	5

6.	**7.**	**8.**	**9.**	**10.**
5682	5648	4793	3875	7392
9	8	7	2	8

Exercise 27. Multiplication

1.	**2.**	**3.**	**4.**	**5.**
8952	6743	7562	8194	6285
39	84	27	65	78

6.	**7.**	**8.**	**9.**	**10.**
4973	1825	7346	5568	7944
63	54	92	87	96

Exercise 28. Multiplication

1.	**2.**	**3.**	**4.**	**5.**
6582	7193	5576	9428	6736
427	836	915	648	257

6.	**7.**	**8.**	**9.**	**10.**
2589	6734	3972	1644	2978
168	395	286	745	896

11.	**12.**	**13.**	**14.**	**15.**
6431	5287	9342	5876	8899
524	638	971	628	759

Additional reserve practice problems in multiplication will be found on page 24.

Exercise 29. Practical Short Methods in Multiplication

1. Multiply 365 by 41.

Regular form:

$$\begin{array}{r} 365\\ 41\\ \hline 365\\ 1460\\ \hline 14965 \end{array}$$

A shorter form:

$$\begin{array}{r} 365\\ 1460\\ \hline 14965 \end{array}$$

2. How many figures are required in the regular form? How many are used in the short form?

3. Using the shorter form, multiply 365 by 31, by 21, by 51, by 71, by 61, by 81, by 91.

4. Multiply 365 by 14.

Regular form:

$$\begin{array}{r} 365\\ 14\\ \hline 1460\\ 365\\ \hline 5110 \end{array}$$

A shorter form:

$$\begin{array}{r} 365\\ 1460\\ \hline 5110 \end{array}$$

In this example, the figure 1 is in tens' place in the multiplier instead of units' place. We may then write down 365 and below it multiply it by 4, placing the 0 in units' place.

5. Using the shorter form, multiply 365 by 18 and by 15.

6. In the same way multiply 866 by 12, by 16 and by 19.

7. A farmer sold a carload of 19 cattle averaging 1475 pounds in weight. How many pounds did the carload of cattle weigh?

8. A truck farmer on a farm of 16 acres made an average profit of $85 per acre. What were his total profits on his farm?

9. Find the cost of a farm of 81 acres at $165 an acre.

10. Make a problem in which this short method can be used. Read it to some member of the class to solve by the short form.

Exercise 30. Multiplication Problems

1. Margaret raised 45 bushels of tomatoes in her garden last summer and sold them at 75 cents a bushel. How much did she receive for her entire crop?

2. Harold raised 62 bushels of onion sets which he sold for 90 cents a bushel. How much did he receive for them?

3. A bushel of potatoes weighs 60 pounds. How many pounds would a wagon load containing 35 bushels weigh?

4. Frank bought a 6-pound roast for 18 cents per pound. Find the cost of the roast.

5. A farmer sold 64 hogs at $25 per head. How much did he receive for them?

6. Edna bought 5 pounds of sugar at 8 cents a pound; 3 pounds of rice at 7 cents a pound; and 3 pounds of apples at 6 cents a pound. Find the total amount of her purchases.

7. Clark's father bought a farm of 80 acres at $135 per acre. Find the cost of the farm.

8. A tourist drove his car at the rate of 15 miles per hour. How far could he travel at that rate in 12 hours?

9. Ruth's father gives her an allowance of 35 cents a week for spending money. How much does she receive in a year of 52 weeks?

10. A stock feeder sold a herd of 23 cattle for $169 per head. How much did the entire herd bring?

11. A sack of wheat holds 2 bushels. A bushel of wheat weighs 60 pounds. How many pounds are there in a wagon load consisting of 20 sacks of wheat?

Exercise 31. Reserve Practice Problems in Multiplication

1. 48×372
2. 29×496
3. 63×581
4. 71×479
5. 58×528
6. 35×613
7. 26×247
8. 97×356
9. 41×168
10. 32×984
11. 17×826
12. 49×573
13. 53×716
14. 64×954
15. 78×832
16. 92×196
17. 89×425
18. 56×789
19. 74×357
20. 23×841
21. 51×236
22. 83×169
23. 62×840
24. 48×205
25. 71×3791
26. 19×2463
27. 37×9524
28. 52×7813
29. 64×5869
30. 91×7148
31. 358×569
32. 274×732
33. 963×146
34. 258×961
35. 702×852
36. 915×347
37. 386×642
38. 471×815
39. 268×793
40. 999×853
41. 137×946
42. 458×275
43. 375×863
44. 246×632
45. 378×519
46. 465×872
47. 927×619
48. 358×824
49. 987×789
50. 536×148
51. 986×2457
52. 654×3682
53. 567×3458
54. 967×2364
55. 985×5729
56. 493×6945
57. 238×7948
58. 689×6197
59. 543×2579
60. 709×5418
61. 667×7963
62. 358×6082
63. 642×7421
64. 159×5809
65. 234×9613
66. 78×35896
67. 56×12768
68. 92×57932
69. 41×48316
70. 73×14695
71. 55×69175
72. 88×48236

73. A barrel of flour weighs 196 pounds. How many pounds of flour are there in 85 barrels?

74. A bushel of shelled corn weighs 56 pounds. How much will a wagon load of 54 bushels of shelled corn weigh?

75. A farmer shipped a load of 95 hogs to market. If the hogs averaged 235 pounds when they were loaded and only 220 pounds when they arrived at the stock yards, how much did they lose in weight?

Milk and Cream Price Card

Milk		Cream	
Pint Bottle	$0.05	½ Pint, Rich	$0.10
Quart Bottle	.09	½ Pint, Whipping	.12
Certified Pint	.08	¼ Pint, Cream	.06
Buttermilk, Quart	.08	Cottage Cheese	.10

Butter 37 cents, 40 cents, 42 cents.

Use the above table in solving the following problems:

Exercise 32

1. What is the cost of two ½ pints of cream and 3 quarts of milk?

2. What do 2 quarts of buttermilk and 2 rolls of cottage cheese cost?

3. How much are 4 pints of milk and two ¼-pint bottles of cream?

4. What is the cost of two ½ pints of cream, 3 packages of cottage cheese, and 2 quarts of milk?

Find the amount of the following weekly dairy orders:

5. Two quarts of milk; three ½-pint bottles of cream; 1 pint of whipping cream; three ½-pint bottles of milk; 3 pounds of butter at 37 cents.

6. Five ½ pints cream; three ½-pint bottles of certified milk; 2 quarts of milk; 4 pounds of butter, 3 pounds at 37 cents and 1 pound at 40 cents.

7. Seven pints of milk; 2 quarts of buttermilk; five ½ pints of cream; 3 pounds of butter at 42 cents.

8. 1 quart of buttermilk; five ½ pints of cream; 3 quarts of milk; 4 pounds of butter at 40 cents.

9. Find the total for the four weeks.

10. Make three problems, using the price card.

Find the amount of the laundry lists in Exercise 33, using the prices given in the following form. Do not use pencil in figuring the total for each item.

Laundry Lists

WHITE STEAM LAUNDRY.
NELSON BROTHERS.

M________________

CALL				DELIVER			
No.	Pieces		Price	No.	Pieces		Price
	Shirts	10			Towels	1	
	" open front				Bath	2	
	pr cuffs	12			Roller	2	
	" pleated				Table Covers	7	
	with cuffs	15			Lunch Cloths	4	
	" work	10			Napkins	1	
	Boys Waists	15			Middies	15	
	Collars	3			Buster Browns	5	
	Cuffs, per pair	5			Wash Cloths	1	
	Handkerchiefs	2			Rough Dry, apiece	3	
	Neckties	5					
	Socks, per pair	5					

Exercise 33

1. 2 work shirts; 4 handkerchiefs; 15 towels; 6 bath towels; 2 lunch cloths; 6 napkins; 2 middies; 6 wash cloths; 12 rough dry.

2. 3 shirts, pleated, with cuffs; 6 handkerchiefs; 7 towels; 8 bath towels; 2 table covers; 1 lunch cloth; 8 napkins; 2 Buster Browns; 4 wash cloths; 18 rough dry.

3. 3 shirts, pleated, with cuffs; 13 collars; 6 handkerchiefs; 2 neckties; 8 towels; 5 bath towels; 1 table cover; 6 rough dry.

4. 2 shirts with cuffs; 2 handkerchiefs; 8 towels; 5 bath towels; 2 lunch cloths; 6 napkins; 6 wash cloths; 24 rough dry.

5. 2 work shirts; 2 with cuffs; 2 boys waists; 6 collars; 3 neckties; 4 pairs socks; 6 bath towels; 10 rough dry.

6. 3 shirts, pleated, with cuffs; 3 shirts with cuffs; 2 work shirts; 2 boy's waists; 8 collars; 5 handkerchiefs; 9 towels; 8 bath towels; 4 roller towels; 2 table covers; 6 napkins; 4 Buster Browns; 2 middies.

7. 2 shirts, pleated, with cuffs; 6 collars; 4 handkerchiefs; 3 neckties; 12 towels; 5 bath towels; 3 roller towels; 8 wash cloths; 8 rough dry.

8. Find the cost of employing a laundress for 6 days at $2.10 (without pencil).

9. Prepare a laundry list including only the articles listed in the sample laundry list. Find the amount of the list at the prices given.

Exercise 34. Division

Practice on the following drill until you can say the correct quotients in 60 seconds or write them in 90 seconds:

$5\overline{)40}$ $2\overline{)10}$ $7\overline{)28}$ $9\overline{)18}$ $6\overline{)36}$ $3\overline{)18}$ $8\overline{)64}$ $5\overline{)35}$ $9\overline{)81}$

$6\overline{)42}$ $4\overline{)8}$ $2\overline{)16}$ $8\overline{)24}$ $3\overline{)27}$ $7\overline{)49}$ $9\overline{)54}$ $7\overline{)56}$ $5\overline{)45}$

$3\overline{)15}$ $8\overline{)48}$ $6\overline{)24}$ $4\overline{)36}$ $2\overline{)14}$ $6\overline{)6}$ $3\overline{)9}$ $8\overline{)72}$ $4\overline{)32}$

$5\overline{)25}$ $2\overline{)6}$ $7\overline{)21}$ $9\overline{)36}$ $4\overline{)16}$ $6\overline{)0}$ $2\overline{)8}$ $9\overline{)9}$ $5\overline{)10}$

$7\overline{)35}$ $8\overline{)32}$ $4\overline{)28}$ $3\overline{)18}$ $9\overline{)45}$ $6\overline{)54}$ $4\overline{)20}$ $8\overline{)8}$ $2\overline{)24}$

Exercise 35. Drill Cards for Division

D	E
8	18
64	72
48	36
16	54
72	126
24	162
88	144
32	108
80	180
40	90

1. Cut a cardboard strip 5 inches by $\frac{3}{4}$ inch.

2. On one side put drill D and on the other side drill E as shown in the pictures at the left.

3. Divide each number in drill D by 2, by 4, by 8.

4. Divide each number in drill E by 3, by 6, by 9.

5. Prepare a drill strip for numbers which contain 5 a whole number of times.

6. Prepare a similar strip for 7.

After the pupils have practiced on the cards with the numbers as indicated, other cards may be made containing a series of numbers such as 48, 54, 30, 72, 16, 24, 40, 60, 56, 12, 32, 45, 36.

Explanation for Short Division

$$\begin{array}{rrl} & 432 & \text{quotient} \\ \text{Divisor} & 7\overline{)3024} & \text{dividend} \end{array}$$

Explanation: 7 is not contained in 3 (thousands). Change the 3 thousands to hundreds, making 30 hundreds. 30 (hundreds) contains 7, 4 (hundred) times, leaving a remainder of 2 (hundreds). Place the 4 in the hundreds' place in the quotient and change the remaining 2 hundreds to tens, making 20 tens. 20 tens+2 tens make 22 tens. 22 (tens) contains 7, 3 (tens) times with a remainder of 1 (ten). Place the 3 in the tens' place in the quotient and change the remaining 1 (ten) to units, making 10 units. 10 units+4 units make 14 units. 14 contains 7, 2 times. Place the 2 in the units' place of the quotient.

Divide and explain the following:

7. 6345÷3
8. 2394÷7
9. 6347[1]÷8
10. 4617÷9
11. 6340÷5
12. 8317÷4
13. 4735÷5
14. 6732÷9

---

[1] The remainder in problem 9 should be expressed in the form $\frac{3}{8}$.

Exercise 36

1. Katherine, Ruth and Henry earned $2.10 from a lemonade stand. How much did they each earn if they divided their profits equally?

2. There are 365 days in a year. This is how many weeks of 7 days each?

3. Nine children together gave $5.40, which they had earned, to the Fresh Air Fund. What was each one's share if they contributed equally to the fund?

4. John received $7.50 in cash for Christmas. His father, mother and uncle each gave him an equal amount. What did each give him?

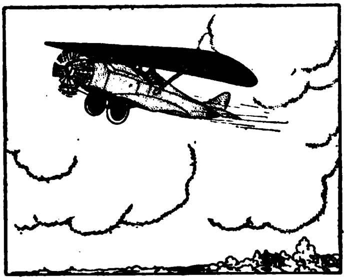
5. 4)6024
6. 5)7580
7. 3)6456
8. 6)2736
9. 5)18279
10. 4)16024
11. 3)34872
12. 6)27942
13. 7)5047
14. 8)34008
15. 7)25928
16. 8)67216
17. 9)47736
18. 7)37856
19. 8)49208
20. 9)83970
21. 6)395412
22. 8)450432
23. 7)294126
24. 9)705618
25. 2)567513
26. 6)364326
27. 9)435816
28. 7)483561
29. 8)143081
30. 3)614322
31. 5)634172
32. 4)834960
33. 9)341937
34. 6)132426
35. 2)843104
36. 8)408264
37. 7)259173
38. 9)235845
39. 3)631413
40. 6)315624

Exercise 37. Long Division

Divide 38778 by 69.

$$\begin{array}{lrl}
 & 562 & \text{quotient} \\
\text{Divisor} & 69\overline{)38778} & \text{dividend} \\
 & \underline{345} & \\
 & 427 & \\
 & \underline{414} & \\
 & 138 & \\
 & 138 &
\end{array}$$

Explanation: 69 is not contained a whole number of times in 38 thousands. To find approximately how many times 69, which is nearly 70, is contained in 387 hundreds, we see that 70 is contained 5 times but not 6 in 387 hundreds. Put the 5 in hundreds' place in the quotient and multiply 69×5 (hundreds), making 345 hundreds. Subtract the 345 hundreds, leaving a remainder of 42 hundreds. Bring down the 7 tens. Proceed in this manner until the units' figure is obtained.



1. In a record flight an aviator traveled in an aeroplane a total distance of about 1320 miles in 24 hours. Find the average speed in miles per hour for this trip.

2. The fastest steamships now cross the Atlantic Ocean, a distance of about 3000 miles, in 5 days. Find the average number of miles per hour traveled by these fast ocean steamships.

3. The yearly rent of a house is $420. What is the rent per month?

4. A class of 25 children together had a sleighing party which cost $5.00. How much did each one have to pay, the expenses being shared equally?

5. There are 324 children in a 12-room school. This is an average of how many in each room?

6. In a 21-room school there are 945 children. This is how many in a room?

7. A wagon load of shelled corn weighed 2240 pounds. How many bushels of corn were there in the load? (1 bushel of shelled corn weighs 56 pounds.)

Solve the following examples in long division:

8. 483÷21
9. 714÷21
10. 903÷21
11. 1176÷21
12. 1344÷21
13. 1538÷21
14. 768÷32
15. 1120÷32
16. 584÷41
17. 792÷33
18. 1683÷33
19. 1075÷43
20. 1836÷51
21. 1612÷62
22. 2272÷71
23. 1378÷53
24. 2646÷63
25. 2646÷63
26. 2555÷73
27. 1014÷39
28. 1888÷59
29. 1200÷48
30. 1768÷68
31. 1258÷37
32. 1322÷42
33. 2881÷67
34. 2088÷87

Exercise 38. Reserve Practice Exercises in Division

A	B	C	D
1. 3261	**6.** 5148	**11.** 91239	**16.** 34765
2. 3072	**7.** 9146	**12.** 20918	**17.** 75969
3. 9206	**8.** 9775	**13.** 18302	**18.** 60898
4. 3710	**9.** 6588	**14.** 46847	**19.** 87879
5. 8049	**10.** 9767	**15.** 57360	**20.** 80798

I. Divide each number in the above list by 2, by 3, by 4, by 5, by 6, by 7, by 8, by 9.

II. Divide each number in columns C and D by 15, by 29, by 37, by 48, by 56, by 74, by 81, by 92.

Exercise 39

Keep a record of your time in working this list of ten long division examples. You should be able to do them in about 20 minutes.

1. 29602÷82
2. 106953÷231
3. 377289÷927
4. 17152÷67
5. 29602÷92
6. 15093÷39
7. 53192÷61
8. 45595÷55
9. 58764÷83
10. 17954÷47

Exercise 40. Reserve Problems in Long Division

1. 1334÷58
2. 1692÷47
3. 9144÷36
4. 26837÷47
5. 57623÷65
6. 20124÷52
7. 67831÷93
8. 29602÷82
9. 57036÷78
10. 63827÷61
11. 564682÷698
12. 672670÷982
13. 599256÷609
14. 621453÷938
15. 803656÷889
16. 300352÷608
17. 410163÷387
18. 561446÷694
19. 224952÷309
20. 645673÷817

Speed Tests

The exercises on pages 33 to 40 and 144 to 152 should now be used at the close of this review to determine whether the class has reached the required standard for this grade. Use the time limits as suggested in these exercises.

If the class is not up to the required standard, the following standardized exercises may be used for practice. For this purpose the order of the examples in each exercise should be frequently changed in order to prevent the pupil from memorizing the answers.

Addition, Subtraction, Multiplication and Division[1]

To the Teacher: The following exercises have been arranged so that the same time should be given for each exercise. For the fifth grade allow $2\frac{3}{4}$ minutes for each exercise. Decrease or increase this standard to meet the capacity of your class but use the same time limit for each of the exercises.

Additional exercises are given on pages 145 to 152 in the second part of the book. These exercises should be given the fifth grade this year and then repeated on a shorter time limit in the sixth grade.

The best results can be secured if it is possible to have these exercises hectographed or mimeographed for drill purposes. Some of the exercises can be conveniently done by placing a sheet of paper under the examples in the book and writing the answers on this sheet of loose paper.

Every exercise should be accurately timed with a watch.

These exercises may be used in two ways:

(1) All pupils may practice on each exercise together. After time is called, the pupils should exchange papers and correct them as the teacher reads the answers. Keep individual records of the number of examples right for each exercise.

(2) Start all of the pupils on Exercise 41. As soon as the pupil has finished correctly all the examples in one exercise within the time allowed, have him try the next exercise. Under this plan a pupil can progress at his own pace. Individual differences in pupils are thus recognized and provided for by this method. The papers of those who have finished should be collected and corrected before the next drill period. Some reliable pupil may be appointed by the teacher to help in this work.

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[1] These exercises were arranged by Charles L. Spain, Assistant Supt. of Schools, Detroit, Michigan.

Exercise 41. Addition

6	9	7	6	2	3	5	1	4	9	5	5
7	1	2	3	1	7	9	0	7	4	4	6
7	7	3	3	8	7	2	4	9	2	9	2

5	9	5	7	2	1	6	1	6	3	3	8
7	6	0	7	8	4	4	9	5	2	6	6
5	9	7	8	7	6	0	9	2	6	4	8

8	9	5	1	1	4	9	4	9	2	4	4
4	5	1	5	8	3	3	1	7	7	0	5
3	3	0	7	2	4	1	0	6	3	5	2

Exercise 42. Subtraction

17	56	98	77	19	34	76	58	49	46	49	65
6	4	7	5	7	4	5	8	3	4	5	3

56	75	97	88	49	83	66	99	89	67	28	35
3	4	6	5	7	2	6	1	5	5	4	3

23	76	83	72	56	67	43	59	66	55	69
2	2	1	2	5	7	3	9	5	5	4

Exercise 43. Addition

4	7	4	1	2	2	5	7	5	3
0	2	4	9	1	4	0	1	4	9
1	3	3	3	8	5	1	3	0	5
3	2	5	0	2	1	1	2	1	8
6	5	8	3	2	6	7	8	1	8

9	2	6	6	7	8	1	3	1
1	6	1	0	9	8	7	5	1
9	5	4	4	8	6	2	8	7
0	4	8	9	6	7	2	3	1
1	6	6	9	5	5	9	3	8

Exercise 44. Addition

55	31	47	24	14	24	32	70
32	48	51	62	15	33	36	18

17	43	20	20	26	81	42	22
72	21	13	50	73	13	27	16

48	33	52	75	32	17	12	26
31	33	41	21	23	62	26	51

Exercise 45. Multiplication

25	46	58	94	87	53	85	67	26	49
2	3	7	5	6	7	4	8	9	5

24	19	28	91	87	39	83	16	54	57
4	2	5	3	7	4	9	7	6	3

Exercise 46. Addition

32	24	31	25	70	42	50	36
16	13	22	33	17	31	30	20
31	62	11	10	11	15	19	32

20	42	25	52	17	10	23	50
19	13	20	15	51	55	61	16
30	22	42	12	20	32	14	12

Exercise 47. Subtraction

68	57	79	32	61	89	75	78	39	48	59	35
27	34	25	21	20	55	60	18	22	36	54	15

44	52	76	53	67	31	44	89	74	42	80	33
31	41	74	33	20	11	22	67	53	32	50	21

The order of the examples in each of these exercises should be frequently changed to prevent memorizing of answers.

Exercise 48. Division

8)48	7)56	9)81	5)45	7)84	4)36	7)63	6)54
5)55	8)56	9)63	7)35	6)60	9)90	4)36	9)45
4)20	7)42	7)70	9)36	8)40	5)35	6)36	6)48
9)99	8)64	9)54	9)27	3)24	6)42	6)66	8)80
7)77	6)72	5)50	3)21	7)28	4)40	5)40	5)25
4)24	3)27	8)96	4)48	3)33	3)30	6)24	8)32
5)15	6)18	7)28	3)36	4)44	2)22	4)16	

Exercise 49. Addition

35	63	48	37	67	38	59	62	67	42
17	27	13	15	26	32	25	18	26	29

49	17	36	27	35	64	34	39	55	24
42	55	15	54	25	26	48	22	18	28

Exercise 50. Addition

5	7	6	9	9	7	1
3	5	1	4	5	7	1
5	4	3	8	7	0	1
1	7	4	5	9	4	0
5	6	9	6	6	9	7
7	8	6	9	1	3	8
7	3	4	1	7	7	9

4	6	9	6	6	4
4	4	6	4	7	5
1	2	4	7	9	1
1	9	8	8	7	8
6	7	2	8	7	6
6	5	5	5	5	7
7	2	7	9	4	8

Exercise 51. Subtraction

56	39	51	45	75	31	92	99
48	28	43	32	54	17	67	75

76	92	29	68	57	44	87	46
25	58	16	49	16	36	68	23

21	74	46	88	62	53	77	55
19	35	12	69	22	34	24	51

Exercise 52. Addition

2	6	5	4	1	3	2	5
3	4	4	1	2	7	5	1
2	4	7	5	3	2	3	9
1	1	3	7	9	7	5	7
4	4	3	3	7	8	1	6
1	4	2	2	8	4	4	7
2	2	5	6	5	2	2	5
5	6	3	4	2	6	3	3
3	2	2	4	3	3	6	1

Exercise 53. Multiplication

158	572	327	689	215	379
4	7	9	3	8	5

427	526	917	669	537	329
3	5	4	2	8	8

Exercise 54. Addition

75	77	85	96	56	89	66	47
38	35	39	34	79	46	54	95

37	46	26	35	32	58	96	34
85	68	75	77	99	47	45	77

Exercise 55. Division

$23\overline{)598}$	$17\overline{)306}$	$72\overline{)864}$
$38\overline{)456}$	$35\overline{)735}$	$27\overline{)729}$

Exercise 56. Addition

58	15	18	22	16	12	52
19	13	47	29	11	25	29
$\underline{20}$	$\underline{32}$	$\underline{20}$	$\underline{20}$	$\underline{66}$	$\underline{37}$	$\underline{14}$
28	50	40	26	25	13	14
22	29	37	17	34	22	28
$\underline{27}$	$\underline{17}$	$\underline{14}$	$\underline{36}$	$\underline{14}$	$\underline{35}$	$\underline{42}$

Exercise 57. Subtraction

63	41	74	63	81	70	84
$\underline{27}$	$\underline{25}$	$\underline{36}$	$\underline{19}$	$\underline{17}$	$\underline{29}$	$\underline{45}$
30	43	78	55	42	63	57
$\underline{22}$	$\underline{39}$	$\underline{49}$	$\underline{37}$	$\underline{17}$	$\underline{44}$	$\underline{48}$
48	41	64	47	40	22	53
$\underline{39}$	$\underline{33}$	$\underline{56}$	$\underline{38}$	$\underline{39}$	$\underline{15}$	$\underline{44}$

Exercise 58. Addition

22	69	35	61	75	37
29	41	36	54	37	54
$\underline{70}$	$\underline{50}$	$\underline{57}$	$\underline{37}$	$\underline{22}$	$\underline{71}$
67	27	53	37	90	31
43	22	75	25	79	57
$\underline{51}$	$\underline{95}$	$\underline{27}$	$\underline{67}$	$\underline{88}$	$\underline{68}$

Exercise 59. Playgrounds

Many city schools and public parks have provided free public playgrounds similar to the one shown in the illustration. How much do you think it costs to buy all the apparatus for one of these playgrounds? The equipment for one of the playgrounds in a city park cost as follows:

2 giant strides	at	$45.00 each
2 parallel bars	at	25.00 each
2 horizontal bars	at	20.00 each
2 lawn swings	at	50.00 each
2 frames of swings	at	65.00 each
4 teeter totters	at	20.00 each
2 slides	at	30.00 each
2 circle bars	at	85.00 each
2 combination frames	at	82.50 each
2 rocking boats	at	85.00 each
2 teeter ladders	at	25.00 each

1. Find the cost of all of the apparatus for this city playground.

2. The playground shown in the picture is 256 feet long and 104 feet wide. How much did the fence on the two ends and one side of this playground cost at $1.00 a foot?

3. A sand pit is provided for the smaller children. How much did 5 loads of sand cost for this pit at $2.50 a load?

4. The average daily attendance at this playground during one year was 183. What was the total attendance for the 185 days that the playground was open?

The children in a small city school decided to equip their playground with apparatus. They found that they could not afford to buy the regular playground apparatus, so they asked the boys in the manual training department to help them make some of their apparatus. They appointed a committee to plan the equipment.

5. The pupils raised their own funds as follows: entertainment, $26.80; bakery sale, $14.25; candy sale, $8.36, and a play, $36.75. Find the total amount which they had with which to equip their playground.

6. The committee decided to buy a slide costing $30.00 and make the rest of the apparatus. How much money did they have left for materials after buying the slide?

7. The manual training class made the teeter totters. They bought the following supplies: 6 heavy planks at 70 cents each; 4 posts at 25 cents each; a 2-inch gas pipe for $1.60, and irons and bolts to fasten the boards to the pipe for $1.25. Find the cost of the teeter totters.

8. A telephone pole was fastened between two trees for a support for the swings. The pole cost $1.25 and the rope for the swings cost $1.85. What was the total cost of the swings?

9. Two sets of basket ball goals were erected at a cost of $10.00 for each pair of goals. How much money was then left for other supplies?

10. Two basket balls were purchased at $6.00 each and the remainder of the money was kept for incidentals and repairs. What sum was left for incidentals and repairs?

CHAPTER II

Factors and Multiples

1. What is the product of 8×12?

8 and 12 are called the *factors* of the product 96.

The word *factor* means *maker.* Two or more numbers, which multiplied together *make* a number, are called *factors* of that number.

2. What is the product of $2 \times 3 \times 5$? What is the product of 6×5? Name other pairs of factors of 30.

The factors 2, 3 and 5 are called *prime* factors of 30. The factor 6 is called a *composite* factor because it may be considered as *composed* of two other factors, 2 and 3.

A *prime number* is a number which has no exact divisor except itself and one. 2, 3, 5 and 23 are examples of prime numbers.

3. Write all the prime numbers between 1 and 50.

A *composite number* is a number which has other exact divisors, except itself and one. 8, 15 and 27 are examples of composite numbers.

4. Write all the composite numbers between 1 and 50.

Exercise 1

Give groups of either prime or composite factors of the following numbers:

1. 12
2. 36
3. 18
4. 40
5. 14
6. 8
7. 63
8. 27
9. 48
10. 81
11. 36
12. 42
13. 56
14. 72
15. 16
16. 28
17. 64
18. 21
19. 9
20. 32

Practice in factoring numbers not only provides excellent training in division, but is also a valuable preparation for the following chapter on fractions. A quick recognition of the factors of a number aids in reducing a fraction to lower terms, in reducing fractions to a common denominator and in cancellation of common factors in multiplication of fractions.

Divisibility of Numbers

It is of great assistance in factoring numbers to be able to tell whether a number is exactly divisible by another number. There are certain short methods for finding whether a number is exactly divisible by certain other numbers.

Exercise 2. Numbers Divisible by 2

1. Write the numbers from 1 to 40.

2. Divide each number by 2 to see which contains 2 exactly.

A number which is exactly divisible by 2 is called an *even* number. A number which is not exactly divisible by 2 is called an *odd* number.

2, 4, 6 and 8 are even numbers and 1, 3, 5, 7 and 9 are odd numbers.

3. Study the numbers you have divided and give in your own words a short way of telling whether a number is exactly divisible by 2.

Without pencil, tell which of the following numbers are exactly divisible by 2:

1. 17
2. 24
3. 39
4. 40
5. 89
6. 96
7. 99
8. 93
9. 104
10. 110
11. 143
12. 187
13. 851
14. 648
15. 920
16. 634
17. 7436
18. 3429
19. 6453
20. 1640

A summary of all the tests of divisibility is given on pages 46 and 47. This summary is given as a check upon the statements formulated by the pupils.

Exercise 3. Numbers Divisible by 4

1. See if the numbers from 1 to 40 which are divisible by 2 are also divisible by 4.

2. See if numbers which end in a digit (figure) that is divisible by 4 are always divisible by 4.

Try such numbers as 244, 314, 128, 518 and 538.

3. See if numbers are divisible by 4 in which the numbers expressed by the last two digits (figures) are always divisible by 4.

Try such numbers as 164, 224, 536, 784, 616 and 896.

4. See if numbers are divisible by 4 in which the numbers expressed by the last two digits are not divisible by 4.

Try such numbers as 314, 561, 826, 935 and 722.

5. Tell in your own words, then, how we can tell which numbers are exactly divisible by 4.

Without pencil, tell which of the following numbers are exactly divisible by 4:

1. 140
2. 622
3. 872
4. 836
5. 918
6. 1564
7. 2853
8. 4626
9. 7948
10. 2328
11. 6346
12. 8038
13. 2718
14. 3352
15. 6644
16. 34512
17. 60376
18. 45125
19. 91604
20. 35136

The term **divisible** in this chapter means **exactly divisible.**

Exercise 4. Numbers Divisible by 8

1. See if numbers are divisible by 8 in which the numbers expressed by the last three digits of each are divisible by 8.

Try such numbers as 4256, 9840, 12928 and 35168.

2. See if numbers are divisible by 8 in which the numbers expressed by the last three digits of each are *not* divisible by 8.

Try such numbers as 3125, 7366, 11274 and 42612.

3. Give a short method, then, for telling when numbers are exactly divisible by 8.

Without pencil, tell which of the following are divisible by 8:

1. 6328
2. 1426
3. 8925
4. 5200
5. 7528
6. 6373
7. 2857
8. 9224
9. 14424
10. 63759
11. 48584
12. 14216
13. 82168
14. 41230
15. 84628
16. 38486

Exercise 5. Numbers Divisible by 3

1. Write all of the numbers from 3 to 144 which contain 3 an integral or whole number of times.

2. Find the sums of the digits in each number. Is each of these sums of the digits exactly divisible by 3?

3. Try other numbers which are not exactly divisible by 3. Are the sums of their digits divisible by 3?

4. What, then, is a short method of finding whether a number is exactly divisible by 3?

Without pencil, tell which of the following numbers are divisible by 3:

1. 4101
2. 2351
3. 8436
4. 7242
5. 4638
6. 3714
7. 5534
8. 7183
9. 5370
10. 6241
11. 8943
12. 1204
13. 4167
14. 9911
15. 6329
16. 8136

Pupils should use short division as a check upon their work in these exercises. This will give added facility in the process of short division.

Exercise 6. Numbers Divisible by 6

1. See if the even numbers in Exercise 5 which are divisible by 3 are also exactly divisible by 6.

2. See if the odd numbers in Exercise 5 which are divisible by 3 are also exactly divisible by 6.

3. Find a short method of finding whether a number is divisible by 6.

Which of the following numbers are divisible by 6?

1. 1422
2. 6342
3. 8535
4. 9487
5. 6384
6. 5631
7. 5734
8. 3744
9. 5837
10. 8316
11. 1806
12. 5891
13. 7458
14. 9864
15. 2968
16. 3807

Exercise 7. Numbers Divisible by 9

The test for divisibility by 9 is very much like that for 3.

1. Write the numbers from 18 to 144 which you know contain 9 an integral number of times.

2. Find the sum of the digits in each number as in numbers divisible by 3.

If you remember the test for 3, you can easily see the test for 9.

3. Tell in your own words the test of divisibility for 9.

Without pencil, tell which of the following are divisible by 9:

1. 234
2. 729
3. 147
4. 279
5. 639
6. 182
7. 175
8. 468
9. 274
10. 936
11. 819
12. 459
13. 4501
14. 6354
15. 9183
16. 2439
17. 1098
18. 2457
19. 3681
20. 4253

21. By adding the digits as you write, make 10 numbers exactly divisible by 9.

The "9" Puzzle

$$\begin{array}{r} 37256 \\ 9 \\ \hline 335304 \end{array}$$

Have a classmate go to the board and multiply any number of 4 or 5 digits by 9. You are to stand with your back to the board so that you can not see his work. Tell him to draw a line through one of the figures of the product. Have him read the other figures slowly so that you can add them. In the illustration the sum of 3, 3, 3, 0 and 4 is 13. We know, by the test for divisibility for 9, that the sum of all of the digits of a number is divisible by 9, if 9 is a factor of the number. The next higher number above 13 that is divisible by 9 is 18. 18—13=5. Therefore you can tell him that he crossed out a 5. This will hold for all digits except 0 and 9. Why can you not distinguish between crossing out a 0 and a 9 in the product?

Try this game with some one who has not played before and see if he can solve the puzzle.

Exercise 8. Numbers Divisible by 5

1. Write the numbers from 1 to 60.

2. Check the ones which contain 5 an integral or whole number of times. Is there anything about their form which helps you to find them?

3. State in your own words the test for divisibility by 5.

Without pencil, tell which of the following are divisible by 5:

1. 65
2. 120
3. 144
4. 172
5. 560
6. 2480
7. 1596
8. 2474
9. 1672
10. 1895
11. 1620
12. 1791
13. 2364
14. 7500
15. 3455
16. 12749
17. 26440
18. 49665
19. 28550
20. 64301

Summary—Tests for Divisibility

1. A number is divisible by 2 if it ends in 0 or any even digit. Such numbers are called *even numbers*.

2. A number is divisible by 3 if the sum of its digits is divisible by 3.

3. A number is divisible by 4 if the number formed by the last two digits is divisible by 4. Only even numbers are divisible by 4.

4. A number is divisible by 5 if it ends in either 0 or 5.

5. Any even number is divisible by 6 if the sum of its digits is divisible by 3.

6. A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

7. A number is divisible by 9 if the sum of its digits is divisible by 9.

Exercise 9. "Buzz"—A Division Drill

Playing "Buzz"

This game is played as follows: Form in a line. Count very rapidly from 1 to 100 by passing from one pupil to another. Suppose that 7 is the "buzz" word. When a number comes to a player that **is exactly divisible by 7 or contains the figure 7,** the player must say "buzz" instead of the number. When 100 is reached begin over again. If a player says 14, 71, or some other number where he should have responded "buzz," the leader should direct him to take his seat. The leader should put a player out if he does not respond quickly by saying either a number or "buzz." Keep counting rapidly until only one player is left.

For example, the counting goes as follows: 1, 2, 3, 4, 5, 6, "buzz," 8, 9, 10, 11, 12, 13, "buzz," 15, 16, "buzz," 18, 19, 20, "buzz," 22, 23, 24, 25, 26, "buzz," "buzz," 29, 30, etc.

This game can be varied by having 6, 8 or 9 for the "buzz" number.

Exercise 10. Multiples

A number is a *multiple* of another number if it is exactly divisible by that number. 8 and 16 are both multiples of 4; 8 being considered as 4 multiplied by 2 and 16 as 4 multiplied by 4.

The number 16 is a multiple of 8 and 2 as well as a multiple of 4.

1. Give all of the multiples of 4 from 4 to 64. State the other factor in each of these multiples of 4.

2. Give all the multiples of 3 from 3 to 36. State the other factor in each multiple.

3. Give the multiples of 5 from 5 to 80; of 6 from 6 to 96; of 7 from 7 to 112; of 8 from 8 to 128; of 9 from 9 to 144. Give the other factor in each multiple.

4. The number 36 is a multiple of seven different numbers besides itself and 1. Name these seven numbers.

Name the different numbers of which each of the following is a multiple:

1. 12
2. 18
3. 14
4. 32
5. 28
6. 15
7. 16
8. 25
9. 21
10. 33
11. 40
12. 35
13. 42
14. 60
15. 72
16. 48
17. 54
18. 63
19. 81
20. 96

Exercise 11. Common Multiples

The number 16 is a multiple of 8. It is also a multiple of 4. 16, then, being a multiple of both 8 and 4, may be said to be a *common* multiple of 8 and 4.

A common multiple of two or more numbers is one that is exactly divisible by each of the numbers.

You will need to be able to recognize a number which is a common multiple of two or more numbers in your study of fractions.

Find, without pencil, the least number which is exactly divisible by each of the following groups of numbers:

1. 2, 3
2. 3, 4
3. 2, 4
4. 4, 8
5. 6, 8
6. 5, 7
7. 3, 5
8. 8, 12
9. 6, 9
10. 6, 14
11. 12, 18
12. 12, 16
13. 8, 9
14. 9, 15
15. 7, 14
16. 14, 21
17. 16, 32
18. 18, 24
19. 16, 20
20. 6, 15
21. 2, 3, 4
22. 2, 4, 8
23. 2, 6, 8
24. 3, 4, 5
25. 5, 6, 2
26. 8, 4, 12
27. 3, 5, 10
28. 7, 14, 21
29. 8, 12, 16
30. 4, 6, 3
31. 2, 4, 10
32. 6, 4, 8
33. 2, 3, 11
34. 4, 6, 9
35. 5, 6, 4
36. 9, 12, 18
37. 12, 15, 30
38. 8, 14, 28
39. 12, 18, 36
40. 12, 24, 36

Since the least number which may be exactly divided by all of the numbers in any one of these groups must be a common multiple of these numbers it is usually called the *least common multiple.*

Exercise 12

Did you have any difficulty in finding the least common multiple in any of the groups in Exercise 11? If so, study this exercise to find an easier way.

1. What are the prime factors of 12? (See page 41 for prime factors.)

2) 12
2) 6
3

Solution: Divide 12 by 2, giving 6 as a quotient. Divide 6 by 2, giving 3 as a quotient. The prime factors of 12= 2×2×3. Use prime numbers (1, 2, 3, 5, etc.) in a similar way in factoring other numbers.

2. What are the prime factors of 36?

3. How many times is 12 contained in 36? Are all of the prime factors of 12 also found in the prime factors of 36?

4. Find the prime factors of 24 and 72. 72 is exactly divisible by 24. Are all of the prime factors of 24 also found in the prime factors of 72?

5. Use other pairs of numbers in which one is exactly divisible by the other until you see clearly the following principle:

If one number contains another number a whole number of times, it also contains all of the factors of that number.

6. Find the least common multiple of 12, 24 and 36.

The least common multiple of these numbers must contain each of these numbers a whole number of times, and hence must contain all of the factors of each number.

The prime factors of 12=2×2×3.

The prime factors of 24=2×2×2×3.

The prime factors of 36=2×2×3×3.

The least common multiple then must contain 2 at least 3 times as a factor in order to include the three twos found as factors in 24. It must also contain two threes in order to include the two threes found as factors in 36.

The least common multiple, then=2×2×2×3×3=72.

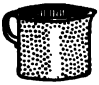
Find the least common multiple of the following numbers by factoring:

7. 12, 16, 32
8. 9, 12, 24
9. 14, 8, 28
10. 16, 24, 36
11. 8, 32, 48
12. 12, 21, 42
13. 6, 12, 30
14. 8, 12, 36
15. 12, 16, 64

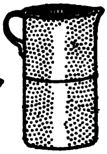
The treatment of the least common multiple has been much simplified in this text book. Its chief use is in connection with finding the least common denominator in fractions. Since most of the denominators in fractions in practical use are small, most of the least common denominators (or multiples) can be easily seen without the necessity of factoring. Have pupils use the factoring method only when they can not readily find the least common multiple without it.

CHAPTER III

FRACTIONS



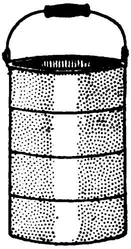
1 Pt.



1 Qt.

How many pint cups of water must be emptied into a quart measure to fill it? If we pour a pint cup full of water into a quart measure, it will be only half filled. A pint is what part of a quart?

Since a pint is $\frac{1}{2}$ of a quart, how many halves are there in a whole quart?



1 Gal.

A quart of water will fill the gallon measure to the lower line. How many quarts are there in a gallon? A quart is what part of a gallon?

$\frac{1}{2}$ of a gallon =how many quarts?

$\frac{1}{2}$ of a gallon $=\frac{?}{4}$ of a gallon?

How many pints are there in a gallon? A pint, then, is what part of a gallon?

How many pints are there in a quart? A quart, then, equals how many eighths of a gallon?

$\frac{1}{4}$ of a gallon $=\frac{?}{8}$ of a gallon?

By comparing pints, quarts and a gallon find the following:

$\frac{1}{2}$ of a gallon $=\frac{?}{8}$ of a gallon?

$\frac{3}{4}$ of a gallon $=\frac{?}{8}$ of a gallon?

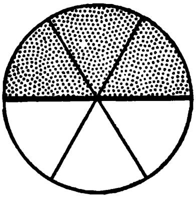
$\frac{2}{4}$ of a gallon $=\frac{?}{8}$ of a gallon?

A gallon $=\frac{?}{4}$ of a gallon? A gallon $=\frac{?}{8}$ of a gallon?

A dime is equal to how many cents? A cent is what part of the value of a dime? A nickel is equal to how many cents? A nickel is equal in value to what part of a dime?

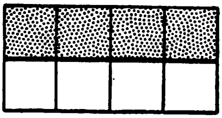
$\frac{1}{2}$ of a dime $= \frac{?}{10}$ of a dime?

Unless a considerable amount of concrete material has been presented to the pupils in studying fractions in the primary book, the teacher should have the pupils use the pint, quart and gallon measures in comparing the above fractional parts.



4. The circle at the left is divided into 6 equal parts. What is each of these 6 parts called? The shaded portion of the circle is what part of the circle?

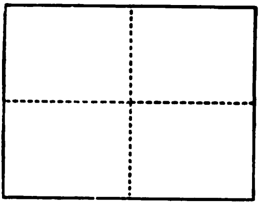
$\frac{1}{2}$ of the circle $= \frac{?}{6}$ of the circle?



5. This rectangle is divided into 8 equal parts. What is each of these parts called? The shaded portion is what part of the retangle?

$\frac{1}{2}$ of the rectangle $= \frac{?}{8}$ of the rectangle?

Take a sheet of paper and fold it so that it is divided into halves and crease. Then fold again and crease. The sheet is now divided into fourths.



$\frac{1}{2}$ of the sheet $= \frac{?}{4}$ of the sheet?

Next fold the sheet once more, thus dividing it into 8 equal parts, or eighths.

Use the sheet of creased paper to answer the following:

$\frac{1}{2}$ of the sheet $= \frac{?}{8}$ of the sheet?

$\frac{1}{4}$ of the sheet $= \frac{?}{8}$ of the sheet?

$\frac{3}{4}$ of the sheet $= \frac{?}{8}$ of the sheet?

Fold the sheet of paper so that it is divided into 16 equal parts. Unfold and use the creased paper to find the following:

$\frac{1}{4}$ of the sheet $= \frac{?}{16}$ of the sheet?

$\frac{2}{4}$ of the sheet $= \frac{?}{16}$ of the sheet?

$\frac{1}{2}$ of the sheet $= \frac{?}{16}$ of the sheet?

$\frac{3}{4}$ of the sheet $= \frac{?}{16}$ of the sheet?

Draw a line a foot long on the board and divide it into 3 equal parts. Each of these parts is called $\frac{1}{3}$ of a foot.

Divide each of the three parts into 2 equal parts. The foot line is now divided into 6 equal parts and each part is called $\frac{1}{6}$ of a foot.

$\frac{1}{3}$ of a foot $= \frac{?}{6}$ of a foot?

$\frac{2}{3}$ of a foot $= \frac{?}{6}$ of a foot?

Comparisons between halves, fourths, eighths and sixteenths may also be made with dry measures: 1 peck=2 gallons=8 quarts=16 pints.

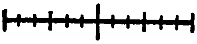
Then 1 pint$=\frac{1}{2}$ quart$=\frac{1}{8}$ gallon$=\frac{1}{16}$ peck.

1 quart$=\frac{?}{4}$ gallon$=\frac{?}{8}$ peck.

This work can then be extended to such fractions as $\frac{3}{4}$ of a gallon, $\frac{3}{8}$ of a gallon, etc.

A foot ruler is divided into how many inches? An inch is what part of a foot?

Draw a line a foot long and mark it off into inches. Draw a longer line through the middle of this foot line dividing it into halves.



$\frac{1}{2}$ of a foot $= \frac{?}{12}$ of a foot?

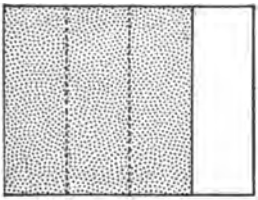
Draw another foot line and mark it off into inches. Draw two longer marks through the four and eight inch marks to divide the line into thirds.

$\frac{1}{3}$ of a foot $= \frac{?}{12}$ of a foot?

Draw a foot line and mark it off into inches. Divide it into fourths.

$\frac{1}{4}$ of a foot $= \frac{?}{12}$ of a foot?

$\frac{3}{4}$ of a foot $= \frac{?}{12}$ of a foot?



In the illustration at the left, the sheet of paper is shown divided into 4 equal parts. The shaded portion of the paper represents three-fourths of the whole sheet. Three-fourths may be written in the shorter form $\frac{3}{4}$. The number $\frac{3}{4}$ is called a *fraction.*[1]

The number 4 (written below the short horizontal line) is called the *denominator* (namer) of the fraction. The denominator shows into how many equal parts the whole thing has been divided. The denominator 4 of the fraction $\frac{3}{4}$ shows that the whole was divided into 4 equal parts.

The denominator also indicates the *size* of the parts. If the sheet were divided into eight equal parts, how would each of the parts compare in size with one of the equal parts if the sheet were divided into fourths? Compare one of the 16 equal parts of the sheet with a fourth and an eighth. We see then that the larger we make the number (as 4, 8 or 16) in the denominator the smaller is the size of each of the equal parts into which the whole has been divided.

The number 3 (written above the line in the fraction $\frac{3}{4}$) is called the *numerator* of the fraction and shows how many of the equal parts have been taken to form the fraction.

---

[1] A fraction is a number composed of one or more of the equal parts of a whole.

Exercise 1

Name the numerators and denominators of the following fractions, telling what each shows, as follows:

1. $\frac{4}{5}$ 5 is the denominator. It shows that the whole thing has been divided into 5 equal parts. 4 is the numerator. It shows that 4 of the 5 equal parts have been taken to form the fraction.

2. $\frac{2}{3}$
3. $\frac{6}{7}$
4. $\frac{5}{8}$
5. $\frac{5}{6}$
6. $\frac{7}{8}$
7. $\frac{5}{9}$
8. $\frac{5}{7}$
9. $\frac{7}{10}$
10. $\frac{8}{12}$
11. $\frac{15}{16}$
12. $\frac{19}{20}$
13. $\frac{8}{9}$
14. $\frac{20}{24}$
15. $\frac{3}{8}$
16. $\frac{1}{2}$
17. $\frac{9}{12}$
18. $\frac{1}{5}$
19. $\frac{32}{40}$
20. $\frac{30}{50}$
21. $\frac{15}{64}$

Exercise 2

A *proper fraction* is one whose value is less than a whole. In a proper fraction the numerator is less than the denominator. $\frac{4}{5}$ is a proper fraction.

An *improper fraction* represents either a whole or more than a whole. The numerator of an improper fraction is equal to or greater than the denominator. $\frac{5}{4}$ is an improper fraction.

A *mixed number* is a number composed of a whole number and a fraction. $1\frac{1}{4}$ is a mixed number.

An improper fraction may be changed to an integer or a mixed number.

1. Change $\frac{9}{3}$ to an integer.

There are 3 thirds in a whole. In 9 thirds there are as many wholes as 3 thirds are contained in 9 thirds. 9 thirds $\div 3$ thirds $= 3$.

The fraction $\frac{9}{3}$ may then be regarded as an indicated division, meaning 9 divided by 3, which equals 3.

Then $\frac{9}{3} = 3$.

2. Change $\frac{39}{8}$ to a mixed number.

$$\frac{39}{8}=39\div8=4\frac{7}{8}.$$

PRINCIPLE: To reduce an improper fraction to an integer or a mixed number divide the numerator by the denominator.

Change the following improper fractions to integers or mixed numbers:

3. $\frac{22}{7}$
4. $\frac{15}{3}$
5. $\frac{17}{5}$
6. $\frac{28}{8}$
7. $\frac{19}{4}$
8. $\frac{29}{9}$
9. $\frac{24}{6}$
10. $\frac{17}{8}$
11. $\frac{9}{4}$
12. $\frac{33}{3}$
13. $\frac{37}{15}$
14. $\frac{41}{7}$
15. $\frac{42}{6}$
16. $\frac{53}{8}$
17. $\frac{7}{2}$
18. $\frac{39}{3}$
19. $\frac{61}{8}$
20. $\frac{25}{5}$
21. $\frac{16}{3}$
22. $\frac{13}{6}$
23. $\frac{48}{6}$
24. $\frac{59}{7}$
25. $\frac{36}{7}$
26. $\frac{29}{3}$
27. $\frac{8}{5}$
28. $\frac{16}{8}$
29. $\frac{41}{5}$
30. $\frac{52}{6}$
31. $\frac{64}{8}$
32. $\frac{65}{9}$
33. $\frac{34}{2}$
34. $\frac{14}{5}$
35. $\frac{12}{4}$
36. $\frac{33}{9}$
37. $\frac{41}{13}$

Exercise 3

1. Change $4\frac{7}{8}$ to an improper fraction.

In 1 whole there are $\frac{8}{8}$. In 4 wholes there are 4 times as many as there are in 1 whole. 4 wholes = 4 times 8 eighths, or $\frac{32}{8}$. $\frac{32}{8}+\frac{7}{8}=\frac{39}{8}$. Therefore, $4\frac{7}{8}=\frac{39}{8}$.

Change the following mixed numbers to improper fractions:

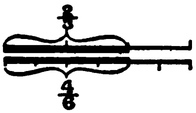
2. $2\frac{1}{3}$
3. $1\frac{4}{5}$
4. $3\frac{5}{7}$
5. $4\frac{3}{4}$
6. $2\frac{1}{2}$
7. $5\frac{3}{5}$
8. $4\frac{3}{8}$
9. $7\frac{1}{3}$
10. $9\frac{3}{5}$
11. $8\frac{2}{3}$
12. $7\frac{2}{5}$
13. $6\frac{4}{9}$
14. $3\frac{1}{7}$
15. $10\frac{3}{4}$
16. $7\frac{5}{6}$
17. $11\frac{1}{3}$
18. $16\frac{1}{2}$
19. $15\frac{3}{8}$
20. $16\frac{2}{3}$
21. $8\frac{7}{8}$
22. $12\frac{4}{5}$
23. $3\frac{11}{12}$
24. $14\frac{1}{5}$
25. $13\frac{3}{4}$
26. $11\frac{4}{5}$
27. $9\frac{3}{7}$
28. $8\frac{4}{7}$
29. $5\frac{1}{2}$
30. $30\frac{1}{4}$
31. $6\frac{1}{4}$
32. $6\frac{3}{11}$
33. $9\frac{7}{8}$
34. $13\frac{3}{5}$
35. $22\frac{1}{5}$
36. $15\frac{2}{3}$
37. $33\frac{1}{2}$

38. Change 5 to thirds; 3 to eighths; 7 to ninths; and 9 to fifths.

39. How many persons will 3 pies serve when cut into sixths?

Exercise 4

The numerator and denominator are called the *terms* of a fraction.



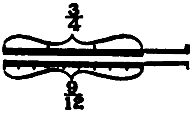
1. Multiply both the numerator and the denominator of the fraction $\frac{2}{3}$ by 2 as follows:

$$\frac{2\times2}{3\times2}=\frac{?}{6}$$

2. How does $\frac{2}{3}$ of the top line in the diagram compare with $\frac{4}{6}$ of the other line?

3. In multiplying the denominator of the fraction $\frac{2}{3}$ by 2 how has the *size* of the equal parts been changed? In multiplying the numerator by 2 how has the *number* of equal parts forming the fraction been changed?

4. If the parts have been made half as large but there are twice as many of them, has the value of the fraction been changed?



5. Multiply both the numerator and the denominator of the fraction $\frac{3}{4}$ by 3. How does $\frac{3}{4}$ of the top line compare with $\frac{9}{12}$ of the lower line in the diagram?

How has the size of the equal parts been affected? How has the number of equal parts forming the fraction been changed? Since the equal parts have been made $\frac{1}{3}$ as large but there are three times as many of them, has the value of the fraction been changed?

6. Multiply both terms of $\frac{5}{6}$ by 4. Draw a figure similar to those in the preceding examples and compare the result with the original fraction.

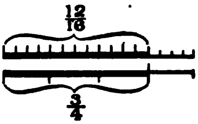
From these examples we see that:

Multiplying both terms of a fraction by the same number does not change the value of the fraction.

Exercise 5

1. Divide both terms of the fraction $\frac{12}{16}$ by 4 as follows:

$$\frac{12 \div 4}{16 \div 4} = \frac{?}{4}$$



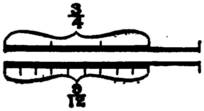
2. Compare the value of $\frac{12}{16}$ of the upper line of the diagram with $\frac{3}{4}$ of the lower line.

3. In dividing the denominator by 4 how has the size of the equal parts been affected?

4. In dividing the numerator by 4 how has the number of parts composing the fraction been changed?

5. Even though there are only $\frac{1}{4}$ as many parts forming the fraction, has the value of the fraction been changed if the equal parts have been made four times as large?

6. Divide both terms of the fraction $\frac{9}{12}$ by 3. How has the size of the equal parts been affected? How has the number of parts forming the fraction been changed? Show why the value of the fraction has not been changed. Also show this by the diagram.



7. Divide both terms of the fraction $\frac{10}{15}$ by 5. Draw a diagram to show that the value of the fraction has not been changed.

From these examples we see that:

Dividing both terms of a fraction by the same number does not change the value of the fraction.

The teacher should supply other examples if necessary to enable the pupils to see clearly how the size and number of the equal parts have been changed.

Exercise 6. Changing to Lowest Terms

We have seen in the preceding exercise that we may divide both the numerator and the denominator of a fraction by the same number without changing the value of the fraction.

The drawing of problem 1 in Exercise 5 showed that $\frac{3}{4}$ of the line was the same as $\frac{12}{16}$ of the line. The terms of the fraction $\frac{3}{4}$ are smaller or lower than the terms of the fraction $\frac{12}{16}$. We change $\frac{12}{16}$ to its lowest terms by dividing both terms by the same number, 4. Or we may say that we cast out a common factor from both numerator and denominator.

A fraction is in its *lowest terms* when there is no number except 1 that is an exact divisor of both terms.

Which is shorter: $\frac{16 \div 2}{32 \div 2} = \frac{8}{16} \div \frac{2}{2} = \frac{4}{8} \div \frac{2}{2} = \frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$
or $\frac{16 \div 16}{32 \div 16} = \frac{1}{2}$?

This illustration shows that we save work in changing a fraction to its lowest terms by dividing both terms by the largest number possible rather than by several smaller numbers.

Change the following to lowest terms:

1. $\frac{6}{8}$
2. $\frac{9}{12}$
3. $\frac{8}{12}$
4. $\frac{6}{9}$
5. $\frac{4}{6}$
6. $\frac{6}{10}$
7. $\frac{6}{12}$
8. $\frac{8}{10}$
9. $\frac{4}{8}$
10. $\frac{5}{10}$
11. $\frac{3}{6}$
12. $\frac{10}{12}$
13. $\frac{7}{14}$
14. $\frac{12}{14}$
15. $\frac{10}{15}$
16. $\frac{8}{16}$
17. $\frac{10}{16}$
18. $\frac{5}{15}$
19. $\frac{12}{18}$
20. $\frac{16}{20}$
21. $\frac{12}{16}$
22. $\frac{14}{21}$
23. $\frac{6}{18}$
24. $\frac{7}{21}$
25. $\frac{20}{22}$
26. $\frac{12}{24}$
27. $\frac{14}{28}$
28. $\frac{15}{30}$
29. $\frac{16}{24}$
30. $\frac{24}{32}$
31. $\frac{24}{28}$
32. $\frac{12}{36}$
33. $\frac{20}{30}$
34. $\frac{18}{36}$
35. $\frac{15}{25}$
36. $\frac{25}{30}$
37. $\frac{24}{40}$
38. $\frac{36}{48}$
39. $\frac{48}{64}$
40. $\frac{125}{200}$

Exercise 7. Adding and Subtracting Like Fractions

1. What is the sum of $4 and $3?

2. What is the sum of 5 books and 3 books?

3. What is the sum of 5 eighths and 2 eighths?

4. What is the sum of $\frac{5}{8}+\frac{2}{8}$?

Notice that in the short form $\frac{5}{8}+\frac{2}{8}$ the denominators merely stand for the name of the equal parts into which the whole has been divided. We are adding 5 eighths and 2 eighths, which equal 7 eighths.

$$\frac{5}{8}+\frac{2}{8}=\frac{7}{8}$$

5. Subtract: $\frac{5}{8}-\frac{2}{8}=$?

In this example we merely subtract the numerators instead of adding them. $\frac{5}{8}-\frac{2}{8}=\frac{3}{8}$.

Add or subtract the following:

Change all answers to lowest terms and change answers that are improper fractions to integers or mixed numbers:

6. $\frac{1}{4}+\frac{3}{4}=$
7. $\frac{3}{16}+\frac{5}{16}=$
8. $\frac{2}{5}+\frac{4}{5}=$
9. $\frac{5}{12}-\frac{2}{12}=$
10. $\frac{3}{7}+\frac{2}{7}=$
11. $\frac{3}{10}+\frac{4}{10}=$
12. $\frac{7}{12}-\frac{6}{12}=$
13. $\frac{5}{9}+\frac{4}{9}=$
14. $\frac{7}{8}-\frac{3}{8}=$
15. $\frac{11}{12}-\frac{7}{12}=$
16. $\frac{3}{4}+\frac{3}{4}=$
17. $\frac{2}{3}+\frac{4}{3}=$
18. $\frac{8}{9}-\frac{5}{9}=$
19. $\frac{8}{11}+\frac{5}{11}=$
20. $\frac{8}{13}+\frac{11}{13}=$
21. $\frac{5}{12}+\frac{1}{12}+\frac{11}{12}=$
22. $\frac{3}{16}+\frac{9}{16}+\frac{11}{16}=$
23. $\frac{2}{5}+\frac{4}{5}-\frac{3}{5}=$
24. $\frac{8}{9}+\frac{2}{9}-\frac{5}{9}=$
25. $\frac{7}{20}+\frac{9}{20}+\frac{19}{20}=$

26. Lucy bought 2 remnants of ribbon. In one there was $\frac{5}{8}$ of a yard and in the other $\frac{7}{8}$ of a yard. How many yards were there in both?

27. John bought $\frac{1}{4}$ of a bushel of onion sets from one dealer and $\frac{3}{4}$ of a bushel from another. How many onion sets did he buy from both?

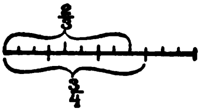
28. Mr. Dean bought the following amounts of seed: $\frac{3}{4}$ pound of onion, $\frac{2}{4}$ pound of carrot, $\frac{3}{4}$ pound of peas, $\frac{3}{4}$ pound of beans and $\frac{1}{4}$ pound cabbage. Find the total weight of these seeds.

Exercise 8

1. What is the sum of 3 books and 4 pencils? We cannot add books and pencils in that form because only like numbers can be added. We must change them to some common name before we can add them:

3 books =3 things
4 pencils =4 things
The sum =7 things.

2. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$? We can not add 2 thirds and 3 fourths because they are not like numbers. We must then change them to a common name. We can change them to twelfths.

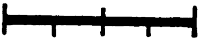


By referring to the diagram, we see that $\frac{2}{3}$ of the line $=\frac{8}{12}$ of the line and $\frac{3}{4}$ of the line $=\frac{9}{12}$ of the line. $\frac{2}{3}+\frac{3}{4}=\frac{8}{12}+\frac{9}{12}$, or $\frac{17}{12}$.

$\frac{8}{12}$ is called an equivalent fraction of $\frac{2}{3}$ because $\frac{8}{12}$ is equal in value to $\frac{2}{3}$. $\frac{9}{12}$ is the equivalent fraction of $\frac{3}{4}$.

In order to add $\frac{2}{3}$ and $\frac{3}{4}$, it was necessary to change them to equivalent fractions with the same denominator (or name). In order to add fractions with different denominators, you will find it a great help to know the common fractional equivalents throughly.

Fractional Equivalents



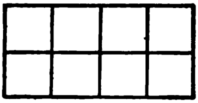
3. By using your ruler or the inch line in the illustration, find how many fourths there are in one-half.

$$\frac{1}{2}=\frac{?}{4}$$

4. Draw a line 6 inches long and divide it into 6 equal parts by marking off inches. Also divide it into halves at the 3-inch mark.

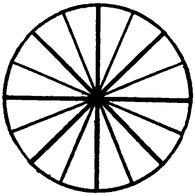
$$\frac{1}{2}=\frac{?}{6}$$

5. The rectangle at the left is divided into how many squares? Each square is $\frac{1}{8}$ of the rectangle. The rectangle is also divided into 4 rows containing 2 squares each. Each row is $\frac{1}{4}$ of the rectangle.



$\frac{1}{4}=\frac{?}{8}$; $\frac{3}{4}=\frac{?}{8}$; $\frac{1}{2}=\frac{?}{8}$

6. This circle is divided into 16 equal parts. The heavy lines divide it into 8 parts.



From this circle find the following fractional equivalents:

$\frac{1}{8}=\frac{?}{16}$; $\frac{2}{8}$ or $\frac{1}{4}=\frac{?}{16}$; $\frac{3}{8}=\frac{?}{16}$; $\frac{4}{8}$ or $\frac{1}{2}=\frac{?}{16}$;
$\frac{5}{8}=\frac{?}{16}$; $\frac{6}{8}$ or $\frac{3}{4}=\frac{?}{16}$; $\frac{7}{8}=\frac{?}{16}$;

Use a foot ruler, yard stick, rectangle and circle to find any of these fractional equivalents which you do not already know:

7.	**8.**	**9.**	**10.**	**11.**	**12.**
$\frac{1}{2}=\frac{?}{4}$	$\frac{1}{2}=\frac{?}{6}$	$\frac{1}{2}=\frac{?}{8}$	$\frac{1}{2}=\frac{?}{10}$	$\frac{1}{2}=\frac{?}{12}$	$\frac{1}{2}=\frac{?}{16}$
$\frac{1}{3}=\frac{?}{6}$	$\frac{1}{4}=\frac{?}{8}$	$\frac{1}{5}=\frac{?}{10}$	$\frac{1}{3}=\frac{?}{12}$	$\frac{1}{4}=\frac{?}{16}$	$\frac{2}{3}=\frac{?}{6}$
$\frac{3}{4}=\frac{?}{8}$	$\frac{2}{5}=\frac{?}{10}$	$\frac{2}{3}=\frac{?}{12}$	$\frac{3}{4}=\frac{?}{16}$	$\frac{3}{5}=\frac{?}{10}$	$\frac{1}{4}=\frac{?}{12}$
$\frac{1}{8}=\frac{?}{16}$	$\frac{4}{5}=\frac{?}{10}$	$\frac{3}{4}=\frac{?}{12}$	$\frac{3}{8}=\frac{?}{16}$	$\frac{1}{6}=\frac{?}{12}$	$\frac{5}{8}=\frac{?}{16}$
$\frac{5}{6}=\frac{?}{12}$	$\frac{7}{8}=\frac{?}{16}$	$\frac{1}{2}=\frac{?}{24}$	$\frac{5}{8}=\frac{?}{24}$	$\frac{5}{6}=\frac{?}{24}$	$\frac{1}{2}=\frac{?}{36}$
$\frac{1}{6}=\frac{?}{36}$	$\frac{1}{4}=\frac{?}{24}$	$\frac{7}{8}=\frac{?}{24}$	$\frac{1}{12}=\frac{?}{24}$	$\frac{1}{4}=\frac{?}{36}$	$\frac{5}{6}=\frac{?}{36}$
$\frac{3}{4}=\frac{?}{24}$	$\frac{1}{3}=\frac{?}{24}$	$\frac{5}{12}=\frac{?}{24}$	$\frac{3}{4}=\frac{3}{36}$	$\frac{1}{9}=\frac{?}{36}$	$\frac{1}{12}=\frac{?}{36}$
$\frac{1}{8}=\frac{?}{24}$	$\frac{2}{3}=\frac{?}{24}$	$\frac{7}{12}=\frac{?}{24}$	$\frac{1}{3}=\frac{?}{36}$	$\frac{3}{8}=\frac{?}{24}$	$\frac{1}{6}=\frac{?}{24}$
$\frac{11}{12}=\frac{?}{24}$	$\frac{2}{3}=\frac{?}{36}$	$\frac{1}{18}=\frac{?}{36}$	$\frac{1}{4}=\frac{?}{32}$	$\frac{3}{4}=\frac{?}{32}$	$\frac{1}{4}=\frac{?}{48}$

Practice on the table of fractional equivalents until you can give the answers rapidly.

Exercise 9. Changing to Higher Terms

In the preceding exercise you have changed fractions to higher terms by reference to figures divided into different parts. The principle found in Exercise 4 may also be used to change fractions to higher terms.

1. Change $\frac{3}{4}$ to twelfths.

The new denominator is 12. The denominator 4 must be multiplied by 3 to make 12. But if the denominator is multiplied by 3, the numerator must also be multiplied by 3, so that the value of the fraction will not be changed.

$\frac{3\times3}{4\times3}=\frac{9}{12}$. Then $\frac{3}{4}$ is equivalent to $\frac{9}{12}$.

Another method may also be used for reducing to higher terms.

2. Change $\frac{2}{3}$ to twelfths.

$\frac{1}{3}=\frac{4}{12}$. $\frac{2}{3}=2\times\frac{4}{12}$, or $\frac{8}{12}$.

Therefore $\frac{2}{3}=\frac{8}{12}$.

Use either of the preceding methods to solve the following:

1. Change $\frac{5}{6}$ to twelfths.

2. Change $\frac{5}{6}$ to forty-seconds.

3. Change $\frac{7}{8}$ to sixteenths.

4. Change $\frac{3}{8}$ to forty-eighths.

5. Change $\frac{2}{3}$ to thirty-sixths.

6. Change $\frac{3}{4}$ to twenty-eighths.

7. Change $\frac{11}{12}$ to thirty-sixths.

8. Change $\frac{5}{9}$ to eighteenths.

9. $\frac{2}{3}=\frac{?}{27}$

10. $\frac{11}{18}=\frac{?}{36}$

11. $\frac{4}{9}=\frac{?}{63}$

12. $\frac{17}{18}=\frac{?}{36}$

13. $\frac{7}{10}=\frac{?}{50}$

14. $\frac{7}{8}=\frac{?}{40}$

15. $\frac{4}{5}=\frac{?}{45}$

16. $\frac{3}{16}=\frac{?}{48}$

17. $\frac{7}{12}=\frac{?}{72}$

18. $\frac{5}{9}=\frac{?}{54}$

19. $\frac{3}{5}=\frac{?}{35}$

20. $\frac{6}{7}=\frac{?}{42}$

21. $\frac{7}{9}=\frac{?}{36}$

22. $\frac{2}{3}=\frac{?}{33}$

23. $\frac{6}{11}=\frac{?}{44}$

24. $\frac{3}{4}=\frac{?}{32}$

Exercise 10. Adding Dissimilar Fractions

Dissimilar fractions are fractions which have different denominators. For example, $\frac{5}{8}$ and $\frac{7}{12}$ are dissimilar fractions.

It was shown in Exercise 8 that dissimilar fractions must be changed to the same denominator before they can be added.

$$\frac{5}{8}+\frac{7}{12}=?$$

What is the least number which will contain 8 and 12 a whole number of times? 24 is the least number (or least common multiple) which will exactly contain both of them. We must then change these fractions to 24ths.

$$\frac{5}{8}=\frac{15}{24}, \text{ and } \frac{7}{12}=\frac{14}{24}$$

Then $\frac{5}{8}+\frac{7}{12}=\frac{15}{24}+\frac{14}{24}=\frac{29}{24}$, or $1\frac{5}{24}$.

Add the following fractions:

1. $\frac{1}{2}+\frac{1}{3}$
2. $\frac{1}{2}+\frac{3}{4}$
3. $\frac{1}{3}+\frac{1}{4}$
4. $\frac{1}{2}+\frac{2}{3}$
5. $\frac{1}{4}+\frac{3}{8}$
6. $\frac{3}{4}+\frac{5}{8}$
7. $\frac{2}{3}+\frac{3}{4}$
8. $\frac{5}{8}+\frac{2}{3}$
9. $\frac{2}{5}+\frac{1}{2}$
10. $\frac{7}{8}+\frac{7}{12}$
11. $\frac{1}{4}+\frac{1}{8}$
12. $\frac{3}{5}+\frac{3}{4}$
13. $\frac{2}{3}+\frac{3}{8}$
14. $\frac{7}{8}+\frac{4}{5}$
15. $\frac{5}{12}+\frac{4}{9}$
16. $\frac{1}{2}+\frac{5}{6}$
17. $\frac{2}{3}+\frac{7}{9}$
18. $\frac{11}{12}+\frac{4}{18}$
19. $\frac{1}{4}+\frac{2}{5}$
20. $\frac{5}{12}+\frac{3}{8}$
21. $\frac{5}{6}+\frac{5}{7}$
22. $\frac{3}{4}+\frac{7}{9}$
23. $\frac{1}{2}+\frac{5}{9}$
24. $\frac{2}{3}+\frac{11}{12}$
25. $\frac{4}{5}+\frac{5}{6}$
26. $\frac{8}{9}+\frac{3}{4}$
27. $\frac{3}{8}+\frac{5}{6}$
28. $\frac{7}{12}+\frac{2}{3}$
29. $\frac{7}{12}+\frac{11}{18}$
30. $\frac{4}{5}+\frac{11}{15}$
31. $\frac{7}{10}+\frac{4}{5}$
32. $\frac{11}{12}+\frac{3}{4}$
33. $\frac{1}{3}+\frac{3}{5}$
34. $\frac{4}{7}+\frac{5}{6}$
35. $\frac{8}{9}+\frac{5}{12}$
36. $\frac{17}{18}+\frac{4}{9}$
37. $\frac{6}{7}+\frac{5}{14}$
38. $\frac{1}{9}+\frac{1}{7}$
39. $\frac{5}{32}+\frac{7}{8}$
40. $\frac{3}{16}+\frac{2}{3}$

Exercise 11. Adding Mixed Numbers

In adding mixed numbers, you will find it convenient to use the column form shown in the illustration.

$$\begin{array}{c|l} 5\frac{1}{2} & \frac{3}{6} \\ 2\frac{2}{3} & \frac{4}{6} \\ \hline 8\frac{1}{6} & \frac{7}{6} \text{ or } 1\frac{1}{6}. \end{array}$$

Add $5\frac{1}{2}$ and $2\frac{1}{3}$. First change the fractions to a common denominator, and then add them. Add the whole numbers, adding in any integer which is left after reducing the sum of the fractions to a mixed number.

Add the following:

1.	2.	3.	4.	5.
$4\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$7\frac{1}{2}$
$2\frac{3}{4}$	$3\frac{5}{8}$	$4\frac{9}{16}$	$2\frac{5}{6}$	$3\frac{7}{8}$

6.	7.	8.	9.	10.
$4\frac{1}{2}$	$3\frac{1}{3}$	$6\frac{1}{4}$	$6\frac{3}{5}$	$7\frac{1}{2}$
$1\frac{7}{12}$	$4\frac{2}{3}$	$2\frac{3}{4}$	$2\frac{2}{3}$	$2\frac{3}{5}$

11.	12.	13.	14.	15.
$6\frac{1}{2}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{3}$	$6\frac{2}{3}$
$2\frac{5}{14}$	$1\frac{8}{9}$	$3\frac{5}{11}$	$2\frac{5}{6}$	$5\frac{3}{4}$

16.	17.	18.	19.	20.
$2\frac{1}{3}$	$5\frac{1}{4}$	$3\frac{1}{2}$	$7\frac{1}{4}$	$10\frac{5}{12}$
$4\frac{5}{6}$	$1\frac{2}{3}$	$5\frac{3}{4}$	$4\frac{3}{5}$	$12\frac{3}{8}$
$3\frac{1}{2}$	$9\frac{1}{6}$	$3\frac{2}{3}$	$6\frac{1}{2}$	$15\frac{2}{3}$

21.	22.	23.	24.	25.
$25\frac{1}{2}$	$14\frac{4}{5}$	$27\frac{1}{4}$	$215\frac{5}{6}$	$233\frac{1}{4}$
$63\frac{5}{6}$	$37\frac{1}{2}$	$67\frac{5}{12}$	$182\frac{2}{3}$	$625\frac{5}{8}$
$45\frac{2}{3}$	$27\frac{7}{10}$	$83\frac{1}{2}$	$158\frac{3}{4}$	$139\frac{1}{2}$
$89\frac{3}{8}$	$54\frac{3}{4}$	$59\frac{4}{9}$	$425\frac{1}{3}$	$647\frac{3}{8}$

Exercise 12

1. For a candy sale some children made $3\frac{1}{2}$ pounds of chocolate fudge, $4\frac{1}{2}$ pounds of peanut brittle and $2\frac{1}{4}$ pounds of taffy. How many pounds did they have to sell?

2. Mary planted in her garden $1\frac{1}{2}$ dozen pink aster plants, $\frac{3}{4}$ dozen purple and $\frac{1}{2}$ dozen white. How many dozen did she plant?

3. Ruth tied a piece of clothes line $1\frac{1}{3}$ yards long to another piece $1\frac{1}{2}$ yards long to make a skipping rope. How many yards long was it? (Allow $\frac{1}{6}$ yard for the knot.)

4. Herbert, Victor and James were one team in a relay race. How many minutes did it take them, together, to run the course if Herbert's share took $\frac{5}{6}$ minute, Victor's $\frac{7}{8}$ minute and James's $\frac{3}{4}$ minute?

5. If Frank studies his arithmetic $\frac{1}{3}$ hour, his spelling $\frac{1}{6}$ hour and works on a geography topic $\frac{1}{2}$ hour, how long has he spent at his lessons?

6. To make a cupboard in which to keep stones and other collections, Robert needs 2 boards $3\frac{1}{2}$ feet long and 3 boards $2\frac{1}{4}$ feet long, all of the same width. From how long a board can these pieces be cut? (Allow $\frac{1}{2}$ inch for waste in sawing.)

7. Henry has in his bank 3 half-dollars and 2 quarters. How many dollars has he?

8. Jane uses $1\frac{1}{2}$ yards of cloth for her apron, $\frac{1}{4}$ yard for a cap and $\frac{1}{8}$ yard for a little bag. How much does she use altogether?

9. In coasting, John's sled goes $1\frac{1}{3}$ feet beyond the record, and the next time goes $1\frac{1}{2}$ feet still farther. By how many feet did he beat the record the second time?

10. Joseph, who weighed $72\frac{1}{2}$ pounds, gained $5\frac{1}{2}$ pounds during the summer vacation on the farm. What did he then weigh?

11. Twenty minutes were added to the afternoon session of a certain school. If it had been $2\frac{1}{3}$ hours before, how long was it after being lengthened?

12. Nancy went shopping and purchased a rutabaga weighing $3\frac{5}{8}$ pounds, a rib roast weighing $4\frac{3}{4}$ pounds and 5 pounds of sugar. Find the total weight of her purchases.

Exercise 13. Subtraction of Fractions

Fractions having different denominators must be changed to the same denominator before they can be subtracted, because only like numbers can be subtracted.

1. Subtract: $\frac{7}{8}-\frac{2}{3}=?$ $\frac{7}{8}=\frac{21}{24}$ and $\frac{2}{3}=\frac{16}{24}$.

$$\frac{7}{8}-\frac{2}{3}=\frac{21}{24}-\frac{16}{24}=\frac{5}{24}.$$

2. $\frac{2}{3}-\frac{1}{2}=?$
3. $\frac{3}{4}-\frac{1}{3}=?$
4. $\frac{7}{8}-\frac{3}{4}=?$
5. $\frac{4}{5}-\frac{2}{7}=?$
6. $\frac{8}{9}-\frac{2}{3}=?$
7. $\frac{5}{6}-\frac{3}{4}=?$
8. $\frac{2}{3}-\frac{3}{5}=?$
9. $\frac{5}{9}-\frac{2}{7}=?$
10. $\frac{11}{12}-\frac{3}{4}=?$
11. $\frac{15}{16}-\frac{3}{8}=?$
12. $\frac{14}{16}-\frac{3}{4}=?$
13. $\frac{7}{8}-\frac{5}{12}=?$
14. $\frac{8}{9}-\frac{3}{4}=?$
15. $\frac{3}{4}-\frac{2}{5}=?$
16. $\frac{29}{32}-\frac{3}{4}=?$
17. $\frac{31}{36}-\frac{2}{3}=?$
18. $\frac{11}{12}-\frac{5}{8}=?$
19. $\frac{7}{8}-\frac{4}{5}=?$
20. $\frac{15}{16}-\frac{1}{2}=?$
21. $\frac{21}{24}-\frac{5}{8}=?$
22. $\frac{15}{16}-\frac{5}{12}=?$
23. $\frac{7}{8}-\frac{3}{7}=?$
24. $\frac{8}{9}-\frac{5}{6}=?$
25. $\frac{14}{15}-\frac{2}{3}=?$

Exercise 14. Subtraction of Mixed Numbers

$$\begin{array}{rr} 11\frac{1}{2} & \frac{4}{8} \\ \underline{5\frac{7}{8}} & \underline{\frac{7}{8}} \\ 5\frac{5}{8} & \end{array}$$

$\frac{1}{2}=\frac{4}{8}$. We can not take $\frac{7}{8}$ from $\frac{4}{8}$, so we take 1 whole unit from the 11 (leaving 10) and change it to eighths. $1=\frac{8}{8}$. $\frac{8}{8}+\frac{4}{8}=\frac{12}{8}$. $\frac{12}{8}-\frac{7}{8}=\frac{5}{8}$. 5 from 10 leaves 5.

Subtract:

1. $\begin{array}{r} 5\frac{1}{2} \\ \underline{2\frac{2}{3}} \end{array}$

2. $\begin{array}{r} 4\frac{1}{2} \\ \underline{1\frac{3}{4}} \end{array}$

3. $\begin{array}{r} 7\frac{1}{2} \\ \underline{3\frac{1}{3}} \end{array}$

4. $\begin{array}{r} 12\frac{3}{8} \\ \underline{7\frac{3}{4}} \end{array}$

5. $\begin{array}{r} 7\frac{1}{4} \\ \underline{2\frac{2}{3}} \end{array}$

6. $\begin{array}{r} 9\frac{3}{8} \\ \underline{4\frac{7}{16}} \end{array}$

7. $\begin{array}{r} 12\frac{1}{5} \\ \underline{6\frac{4}{5}} \end{array}$

8. $\begin{array}{r} 4\frac{1}{4} \\ \underline{3\frac{7}{8}} \end{array}$

9. $\begin{array}{r} 12\frac{5}{6} \\ \underline{4\frac{1}{3}} \end{array}$

10. $\begin{array}{r} 9\frac{5}{8} \\ \underline{6\frac{1}{4}} \end{array}$

11. $\begin{array}{r} 8\frac{1}{3} \\ \underline{3\frac{5}{5}} \end{array}$

12. $\begin{array}{r} 10\frac{5}{24} \\ \underline{6\frac{11}{12}} \end{array}$

13. $\begin{array}{r} 12\frac{1}{3} \\ \underline{5\frac{5}{6}} \end{array}$

14. $\begin{array}{r} 9\frac{1}{2} \\ \underline{7\frac{7}{12}} \end{array}$

15. $\begin{array}{r} 4\frac{3}{4} \\ \underline{3\frac{2}{3}} \end{array}$

Exercise 15. Subtraction of Fractions

1. It is $2\frac{1}{2}$ miles to Crow's Island from Jack's house. When the boys have walked $1\frac{1}{4}$ miles, how many miles have they yet to go?

2. Raymond's market basket, filled, weighed $12\frac{1}{2}$ pounds. When John took out a package of meat weighing $2\frac{1}{4}$ pounds, how many pounds was Raymond carrying?

3. William had $\$1\frac{1}{2}$. He spent $\$\frac{3}{4}$. How much had he left?

4. Bertha and Paul can not spend their pennies for cheap candy, but every week their father brings them 2 pounds of good candy. If they, together, eat $\frac{3}{4}$ pound the first day, how much is left to last the rest of the week?

5. Richard saws a piece $12\frac{1}{4}$ inches long from a board 26 inches long. How long a piece remains? (Allow $\frac{1}{8}$ of an inch for the cut of the saw.)

6. Louise's mother gave her $1\frac{1}{8}$ yards of pink gingham. Louise used $\frac{3}{4}$ yard for a doll dress. How long a piece remained?

7. A certain village is $17\frac{1}{2}$ miles from a large city. When a train has traveled $6\frac{1}{2}$ miles toward the city, how many miles has it yet to go?

8. Charles cuts a slip of cardboard $6\frac{1}{8}$ inches wide from a piece 27 inches in width. How wide is the piece remaining?

9. Marion planted 2 dozen crocus bulbs in pots. When $\frac{3}{4}$ dozen had bloomed, how many were not blooming?

10. The morning session in a certain school was $2\frac{3}{4}$ hours long; the afternoon session was $2\frac{1}{3}$ hours. What was their difference in length?

11. Once a week there is $\frac{1}{2}$ hour for morning exercise attended by all the classes. How much of the morning session is then left for the regular work?

Exercise 16. Review Problems in Fractions

1. What waste is there in sawing from a 6-foot board a piece $2\frac{1}{2}$ feet long and one $3\frac{1}{4}$ feet? How much of this waste would result from the sawing if the cut of the saw was $\frac{1}{16}$ inch wide?

2. Jane has $\$\frac{1}{4}$ and $\$\frac{1}{2}$ and wants to get a skating cap and a scarf which together cost $\$1\frac{1}{4}$. How much more money does she need?

3. There are four dozen chair desks in a school room. When $1\frac{1}{2}$ dozen have been put in a semicircle and $\frac{3}{4}$ dozen in each of 3 rows behind the semicircle, what part of a dozen remains?

4. Helen's mother made $2\frac{1}{2}$ dozen sugar cookies and 4 dozen molasses cookies. She gave Helen $\frac{1}{2}$ dozen of each for a tea party. How many were then left?

5. Mabel's class had $\frac{1}{2}$ hour study period. If Mabel finished her arithmetic in $\frac{1}{4}$ hour and learned her spelling words in $\frac{1}{12}$ hour, how much time had she left to read a book?

6. Hazel and Ruth made 2 dozen pieces of white fudge and $3\frac{1}{4}$ dozen of brown. They ate 3 pieces each. How many dozen had they left to fill boxes?

7. Rose has a pretty piece of ribbon $4\frac{1}{2}$ yards long. How long a sash can she have after she has cut off two hair ribbons each $\frac{5}{6}$ yard long?

8. Marjorie counted the pansies in her garden and found 6 dozen blossoms. How many were left in the garden when she had picked 1 dozen blue ones, $\frac{1}{2}$ dozen yellow, and $2\frac{3}{4}$ dozen mixed colors?

9. A picture $5\frac{1}{2}$ inches long is to be mounted so as to have a $1\frac{1}{2}$-inch margin at the top and a $2\frac{1}{2}$-inch one at the bottom. How much must be trimmed from the mounting cardboard, which is 10 inches long?

10. What waste is there in cutting 2 strips $\frac{7}{8}$ foot wide from a sheet of cardboard 3 feet wide?

11. Stuart had $\$\frac{3}{4}$ to spend on an excursion. His fare was $\$\frac{1}{2}$; he spent a dime riding on the merry-go-round and a nickel ($\$\frac{1}{20}$) for an ice-cream cone. What part of a dollar had he left?

Exercise 17. Multiplying an Integer by a Fraction

1. There are 16 ounces in a pound. How many ounces are there in $\frac{1}{8}$ of a pound?

In order to find $\frac{1}{8}$ of 16 ounces, we must divide 16 ounces into 8 equal parts. $16 \div 8 = 2$.

2. How many ounces are there in $\frac{3}{8}$ of a pound?

Since there are 2 ounces in 1 eighth of a pound, in 3 eighths of a pound there are 3×2 ounces, or 6 ounces.

3. Find $\frac{3}{4}$ of 28.

$\frac{1}{4}$ of $28 = 28 \div 4$ or 7. $\frac{3}{4}$ of $28 = 3 \times 7 = 21$.

4. $\frac{2}{3}$ of $36 =$?

5. $\frac{7}{8}$ of $64 =$?

6. $\frac{5}{6}$ of $30 =$?

7. $\frac{4}{5}$ of $55 =$?

8. $\frac{8}{9}$ of $81 =$?

9. $\frac{5}{7}$ of $56 =$?

10. $\frac{3}{8}$ of $160 =$?

11. $\frac{7}{12}$ of $48 =$?

12. $\frac{3}{4}$ of $200 =$?

13. $\frac{5}{9}$ of $72 =$?

14. $\frac{3}{7}$ of $63 =$?

15. $\frac{5}{8}$ of $32 =$?

16. Find the cost of $\frac{2}{3}$ pound of candy at 45 cents a pound.

17. Find the cost of $\frac{3}{4}$ pound of meat at 28 cents a pound.

18. Find the cost of $\frac{2}{3}$ of a dozen of oranges at 30 cents a dozen?

19. John helps pass geographies. If each weighs $\frac{7}{8}$ pound, how many pounds are there in a load if he takes 6 at one time?

20. Margaret has an allowance of $\$\frac{1}{4}$ a week. If she saves it all for 5 weeks, how much money has she?

21. After a candy sale, a class, besides smaller coins, had three $\$\frac{1}{2}$ and five $\$\frac{1}{4}$. How many dollars had they in these coins?

22. Joseph, who lives $\frac{3}{4}$ mile from school, walks this distance four times a day. How far does he walk in going back and forth to school in a week?

23. May practices on the piano $\frac{1}{2}$ hour every day except Sunday. How many hours does she practice in a week?

24. Father bought 2 tickets to the circus which cost $\$\frac{1}{2}$ and three which cost $\$\frac{1}{4}$. What did the five tickets cost?

25. How much ice cream is needed at a party for 18 children, allowing $\frac{1}{6}$ quart apiece?

Exercise 18. Multiplying an Integer by a Mixed Number

1. Multiply 23 by $3\frac{1}{2}$.

$$\begin{array}{l} 23 \\ \underline{3\frac{1}{2}} \\ 69 \\ \underline{11\frac{1}{2}} \\ 80\frac{1}{2} \end{array}$$

First multiply 23 by 3, which equals 69. Then find $\frac{1}{2}$ of 23, which equals $11\frac{1}{2}$. Then add the two partial products together, $69+11\frac{1}{2}=80\frac{1}{2}$. Therefore $3\frac{1}{2}\times23=80\frac{1}{2}$.

Multiply:

2. $2\frac{1}{2}\times17$

3. $5\frac{3}{5}\times35$

4. $7\frac{1}{2}\times25$

5. $5\frac{3}{4}\times20$

6. $4\frac{2}{3}\times15$

7. $6\frac{7}{8}\times32$

8. $2\frac{5}{9}\times18$

9. $8\frac{1}{3}\times22$

10. $3\frac{1}{7}\times12$

11. $4\frac{3}{5}\times40$

12. $11\frac{1}{3}\times32$

13. $16\frac{2}{3}\times6$

14. $25\frac{1}{2}\times13$

15. $42\frac{1}{5}\times24$

16. $16\frac{1}{3}\times18$

17. Find the cost of 3 yards of gingham at $\$0.12\frac{1}{2}$ per yard.

18. How long a piece of cord must Grace buy to finish 2 calendars, if each one requires $6\frac{1}{2}$ inches?

19. How much of a 10-yard bolt of baby ribbon does Amy have left after tying up 3 packages, $2\frac{1}{4}$ feet being needed for each one?

20. Betty made 4 aprons just alike except for trimming, each one requiring $1\frac{1}{4}$ yards. How much material was needed?

21. The children in a vacation school made some khaki bags to carry with them on their hikes. Each bag required $1\frac{1}{8}$ yards. How many yards were needed to supply a class of 16, not allowing any for waste?

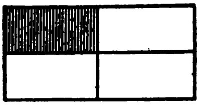
22. A rod is $16\frac{1}{2}$ feet long. Express in feet a distance of 10 rods.

23. A rod is $5\frac{1}{2}$ yards long. How many yards of fencing are needed for a garden measuring 5 rods around?

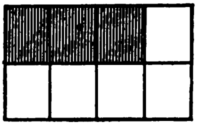
24. Each of 18 members of a graduating class bought $1\frac{1}{4}$ yards of ribbon for class colors. How many yards did they buy? How much did all of the ribbon cost at 20 cents a yard?

25. Find the cost of $1\frac{5}{8}$ pounds of steak at 24 cents a pound.

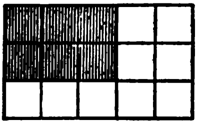
Exercise 19. Multiplying a Fraction by a Fraction



1. In the rectangle the line drawn through the middle divides the rectangle into halves. The shaded portion represents $\frac{1}{2}$ of $\frac{1}{2}$. What is $\frac{1}{2}$ of $\frac{1}{2}$ of the rectangle?



2. The vertical lines divide this rectangle into fourths. The shaded portion represents $\frac{1}{2}$ of $\frac{3}{4}$ of the rectangle. What part of the rectangle does $\frac{1}{2}$ of $\frac{3}{4}$ of the rectangle represent?



3. The vertical lines in the rectangle divide it into fifths. The shaded portion represents $\frac{2}{3}$ of 3 of these fifths. What part of the whole rectangle does $\frac{2}{3}$ of $\frac{3}{5}$ of the rectangle represent?

Draw rectangles to illustrate the following:

4. $\frac{1}{2}$ of $\frac{1}{3}$ = ?
5. $\frac{2}{3}$ of $\frac{3}{4}$ = ?
6. $\frac{1}{4}$ of $\frac{1}{3}$ = ?
7. $\frac{1}{2}$ of $\frac{4}{7}$ = ?
8. $\frac{2}{3}$ of $\frac{4}{5}$ = ?
9. $\frac{1}{5}$ of $\frac{2}{3}$ = ?
10. $\frac{3}{5}$ of $\frac{3}{4}$ = ?
11. $\frac{3}{4}$ of $\frac{2}{3}$ = ?
12. $\frac{3}{8}$ of $\frac{1}{2}$ = ?
13. $\frac{1}{2}$ of $\frac{5}{6}$ = ?
14. $\frac{1}{3}$ of $\frac{3}{5}$ = ?
15. $\frac{3}{8}$ of $\frac{2}{3}$ = ?
16. $\frac{1}{2}$ of $\frac{5}{8}$ = ?
17. $\frac{1}{3}$ of $\frac{7}{8}$ = ?
18. $\frac{3}{4}$ of $\frac{7}{8}$ = ?
19. $\frac{1}{2}$ of $\frac{7}{12}$ =?
20. $\frac{1}{4}$ of $\frac{3}{5}$ = ?
21. $\frac{2}{3}$ of $\frac{5}{6}$ = ?
22. $\frac{3}{5}$ of $\frac{3}{8}$ = ?
23. $\frac{2}{3}$ of $\frac{8}{9}$ = ?
24. $\frac{7}{8}$ of $\frac{6}{7}$ = ?

The word "of" may be replaced by the sign ×.

The rectangle shows that $\frac{2}{3}\times\frac{3}{5}=\frac{6}{15}$.

How does the numerator of the result compare with the product of the numerators of the fractions? How does the denominator of the result compare with the product of the denominators of the fractions?

Answer the same questions for enough of the above problems to see clearly the

> **PRINCIPLE: In multiplying one fraction by another, the numerator of the result is the product of the numerators of the two fractions and the denominator of the result is the product of the denominators of the two fractions.**

Exercise 20

In the previous exercise we found that $\frac{2}{3}\times\frac{3}{5}=\frac{6}{15}$. The product $\frac{6}{15}$ must then be changed to lowest terms by dividing both terms by 3, making $\frac{2}{5}$.

We see in the expression $\frac{2}{3}\times\frac{3}{5}$ that 3 is a factor of both the numerator and the denominator. We may then divide the product of the numerators and the product of the denominators by 3 before actually multiplying.

$$\frac{2}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{3}}}{5}=\frac{2}{5}.$$

This process of dividing one factor of the numerator and one factor of the denominator before multiplying is called *cancellation.* Cancellation, then, is really a method of changing to lower terms before multiplying.

Multiply the following, using cancellation whenever possible:

1. $\frac{2}{3}\times\frac{3}{4}=$?
2. $\frac{5}{6}\times\frac{9}{11}=$?
3. $\frac{2}{7}\times\frac{2}{3}=$?
4. $\frac{3}{12}\times\frac{4}{5}=$?
5. $\frac{2}{9}\times\frac{3}{7}=$?
6. $\frac{4}{11}\times\frac{15}{16}=$?
7. $\frac{3}{8}\times\frac{4}{5}=$?
8. $\frac{24}{25}\times\frac{3}{4}=$?
9. $\frac{5}{12}\times\frac{13}{15}=$?
10. $\frac{15}{16}\times\frac{4}{5}=$?
11. $\frac{3}{5}\times\frac{4}{7}=$?
12. $\frac{2}{3}\times\frac{5}{6}=$?
13. $\frac{8}{9}\times\frac{15}{16}=$?
14. $\frac{12}{14}\times\frac{28}{36}=$?
15. $\frac{3}{4}\times\frac{7}{8}=$?
16. $\frac{5}{9}\times\frac{12}{16}=$?
17. $\frac{8}{15}\times\frac{5}{12}=$?
18. $\frac{3}{7}\times\frac{35}{36}=$?
19. $\frac{3}{7}\times\frac{5}{6}=$?
20. $\frac{11}{12}\times\frac{2}{3}=$?
21. $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=$?
22. $\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}=$?
23. $\frac{4}{7}\times\frac{3}{4}\times\frac{14}{15}=$?
24. $\frac{10}{12}\times\frac{3}{5}\times\frac{7}{9}=$?
25. $\frac{1}{2}\times\frac{1}{3}\times\frac{3}{5}=$?
26. $\frac{15}{16}\times\frac{12}{20}\times\frac{18}{24}=$?
27. $\frac{5}{8}\times\frac{11}{12}\times\frac{4}{6}=$?
28. $\frac{3}{5}\times\frac{7}{8}\times\frac{4}{9}=$?
29. $\frac{1}{3}\times\frac{6}{7}\times\frac{21}{24}=$?
30. $\frac{2}{5}\times\frac{3}{4}\times\ =\frac{1}{12}$?

31. Peter bought $\frac{3}{4}$ pound of notebook paper for his sister Edna and himself. If the paper was equally divided, what part of a pound had each? $\frac{1}{2}\times\frac{3}{4}=?$

32. Harry's mother cut $\frac{1}{2}$ pie into 4 equal pieces for Harry and 3 playmates. Each had what part of a pie? $\frac{1}{4}\times\frac{1}{2}=?$

33. Marion made 6 badges for her club out of $\frac{1}{2}$ yard of ribbon. How long was each badge? $\frac{1}{6}\times\frac{1}{2}=?$

34. One-third of a $\frac{3}{4}$-hour recitation period is used for study. The study period is what part of an hour long?

35. George roller-skates $\frac{1}{2}$ mile from his home to school. One day a strap broke and he went $\frac{1}{3}$ of the distance on one skate. What part of a mile was this?

Exercise 21. Multiplying a Mixed Number by a Fraction or a Mixed Number

You have already learned how to multiply a fraction by a fraction. If you have a mixed number to be multiplied by a fraction, or another mixed number, you will find it much easier in most cases to reduce the mixed numbers to improper fractions and then multiply. Use cancellation whenever possible.

1. Multiply $3\frac{1}{3}\times 2\frac{3}{5}$. Change the mixed numbers to improper fractions: $3\frac{1}{3}\times 2\frac{3}{5}=\frac{\overset{2}{\cancel{10}}}{3}\times\frac{13}{\underset{1}{\cancel{5}}}=\frac{26}{3}=8\frac{2}{3}$.

Multiply the following:

2. $1\frac{1}{2}\times 2\frac{2}{5}$
3. $3\frac{2}{3}\times 5\frac{1}{4}$
4. $7\frac{1}{2}\times 8\frac{2}{5}$
5. $1\frac{3}{4}\times \frac{8}{9}$
6. $\frac{3}{4}\times 1\frac{3}{5}$
7. $5\frac{1}{3}\times 3\frac{3}{4}$
8. $5\frac{1}{2}\times 5\frac{1}{2}$
9. $\frac{7}{8}\times 3\frac{1}{5}$
10. $2\frac{1}{3}\times 3\frac{6}{7}$
11. $5\frac{1}{4}\times 2\frac{1}{3}$
12. $1\frac{7}{8}\times 3\frac{2}{3}$
13. $9\frac{1}{4}\times 1\frac{1}{2}$
14. $\frac{2}{3}\times 2\frac{3}{8}$
15. $5\frac{1}{6}\times 2\frac{2}{5}$
16. $8\frac{2}{3}\times 9\frac{3}{5}$
17. $3\frac{1}{7}\times 2\frac{1}{2}$
18. $7\frac{1}{2}\times 4\frac{2}{5}$
19. $\frac{15}{16}\times 3\frac{3}{5}$
20. $5\frac{1}{3}\times 6\frac{1}{2}$
21. $8\frac{3}{4}\times \frac{2}{3}$
22. $16\frac{2}{3}\times \frac{7}{8}$

23. Madeline's mother made $8\frac{1}{2}$ dozen glasses of jelly. One-half of this number was apple. How many dozens were apple?

24. One-third of the $6\frac{1}{2}$ dozen was grape? How many dozen glasses of grape jelly were there?

25. Marie bought $1\frac{1}{2}$ pounds of brown sugar and used $\frac{1}{2}$ of the amount bought for making candy. How much did she use?

26. In making a window box, James sawed a board $6\frac{1}{2}$ feet long into 2 equal lengths. How long was each half?

27. Ethel cut a piece of net $2\frac{1}{2}$ feet long into curtains for her doll house. How long was each piece if she used $\frac{1}{5}$ of the whole length for each curtain?

28. Find the cost of $2\frac{1}{2}$ yards of gingham at \$0.12$\frac{1}{2}$ a yard.

29. What is the cost of $1\frac{1}{2}$ dozen napkins at \$4$\frac{1}{2}$ a dozen?

Exercise 22. Making Candy

These three girls decided to make candy for their Christmas presents. Each girl brought a recipe which she had tried before.

Mabel's Recipe for Chocolate Nut Fudge

(Makes about 40 pieces ½ inch thick)

1½ cups granulated sugar
½ cup light brown sugar
½ cup grated chocolate
¾ cup milk
butter, size of a walnut
½ cup chopped nut meats
vanilla

Boil sugar, chocolate and milk until the candy, when dropped in water, makes a soft ball. Take from stove, add butter and set the pan to cool in water. Stir until thick; add flavoring and nuts, and pour into buttered tin.

1. The girls decided to make enough fudge for 20 pieces each. Find the amounts of each ingredient which they would need to make that amount.

Eleanor's Recipe for Peanut Brittle

½ cup granulated sugar
4½ cups shelled peanuts rolled under rolling pin

Stir sugar in a saucepan on the stove until melted; put in the peanuts, stir quickly, and pour into a buttered dish.

2. Each girl made this candy by herself and made ½ of the recipe. Find the amount of each ingredient used by each girl.

3. Find the amount of each of the ingredients used by all 3 girls.

Helen's Recipe for Chocolate Caramels

$\frac{1}{4}$ pound grated bitter chocolate
$\frac{1}{2}$ cup butter
$2\frac{1}{4}$ cups brown sugar
$\frac{1}{2}$ cup cream

4. The girls decided to make twice this recipe. Find the amount of each ingredient used.

Exercise 23. Dividing a Fraction by an Integer

If we divide $\frac{3}{4}$ of an inch as shown in the figure by 2 (or $\frac{2}{1}$), each part is only $\frac{1}{2}$ as great.

Then $\frac{3}{4} \div 2 = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

Make a drawing to show that $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

In each of these cases the numerator has remained the same, but the denominator has been multiplied by the integer.

Make a drawing to show that $\frac{2}{3} \div 2 = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$.

Make a drawing to show that $\frac{4}{5} \div 2 = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$.

In each of these two cases the denominators have remained the same, but the numerators have been divided by the integer 2.

Use other fractions and other integers if necessary to show the

PRINCIPLE: If a fraction is to be divided by an integer, if possible divide the numerator of the fraction by the integer, otherwise multiply the denominator of the fraction by the integer.

Divide:

1. $\frac{5}{6} \div 3$
2. $\frac{4}{15} \div 2$
3. $\frac{4}{7} \div 2$
4. $\frac{11}{12} \div 11$
5. $\frac{3}{4} \div 5$
6. $\frac{9}{16} \div 3$
7. $\frac{7}{9} \div 2$
8. $\frac{4}{7} \div 4$
9. $\frac{3}{5} \div 4$
10. $\frac{6}{8} \div 3$
11. $\frac{8}{9} \div 3$
12. $\frac{10}{12} \div 5$
13. $\frac{7}{8} \div 2$
14. $\frac{8}{9} \div 4$
15. $\frac{5}{8} \div 3$
16. $\frac{5}{6} \div 2$
17. $\frac{7}{24} \div 7$
18. $\frac{6}{11} \div 3$
19. $\frac{2}{3} \div 3$
20. $\frac{5}{6} \div 5$

21. Three pupils buy together $\frac{3}{4}$ pound of notebook paper. What part of a pound is each one's share?

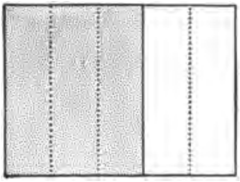
22. Joseph cut 8 score cards from $\frac{1}{2}$ sheet of card board. What part of a sheet was each score card?

23. Mary cut $\frac{3}{4}$ cake into 12 pieces. Each piece was what part of the whole cake?

24. Jean used $\frac{1}{2}$ of an opened can of enamel, $\frac{1}{3}$ full, in painting her golf balls. What part of a can of enamel did she use?

25. Majorie took out $\frac{1}{3}$ of a box of raisins $\frac{3}{4}$ full to use for cookies. What part of a box did she use?

Exercise 24. Dividing an Integer by a Fraction



In the illustration of a sheet of paper at the left, the shaded portion is what fraction of the whole sheet of paper?

The whole sheet is divided into how many fifths?

The fraction 3 fifths is contained in the whole sheet (5 fifths) how many times?

Suggestion: The remainder, 2 fifths, is what part of the fraction 3 fifths?

From this illustration we see that the fraction $\frac{3}{5}$ is contained in the whole sheet (or $\frac{5}{5}$) $1\frac{2}{3}$ or $\frac{5}{3}$ times.

Then $1 \div \frac{3}{5} = \frac{5}{3}$.

1. What is $2 \div \frac{3}{5}$?

$1 \div \frac{3}{5} = \frac{5}{3}$. Then $2 \div \frac{3}{5} = 2 \times \frac{5}{3} = \frac{10}{3}$, or $3\frac{1}{3}$.

2. What is $5 \div \frac{2}{3}$?

Draw a figure similar to the illustration above to show that $1 \div \frac{2}{3}$ is $\frac{3}{2}$.

Then $5 \div \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2}$, or $7\frac{1}{2}$.

3. What is $3 \div \frac{3}{4}$?

Draw a figure to show that $1 \div \frac{3}{4} = \frac{4}{3}$.

Then $3 \div \frac{3}{4} = 3 \times \frac{4}{3} = 4$.

We see that in each of these examples, the division example has been changed into a multiplication example. In the illustration $2 \div \frac{3}{5} = 2 \times \frac{5}{3}$, the divisor has become *inverted*, the denominator of $\frac{3}{5}$ becoming the numerator of $\frac{5}{3}$ and the numerator of $\frac{3}{5}$ becoming the denominator of $\frac{5}{3}$.

Thus we see the

PRINCIPLE: To divide an integer by a fraction, multiply the integer by the fraction inverted.

Divide the following:

1. $1 \div \frac{3}{4}$
2. $2 \div \frac{2}{3}$
3. $1 \div \frac{7}{8}$
4. $3 \div \frac{4}{5}$
5. $4 \div \frac{1}{2}$
6. $9 \div \frac{1}{3}$
7. $7 \div \frac{3}{5}$
8. $6 \div \frac{5}{6}$
9. $5 \div \frac{2}{5}$
10. $8 \div \frac{5}{8}$
11. $7 \div \frac{7}{8}$
12. $4 \div \frac{5}{9}$
13. $5 \div \frac{3}{7}$
14. $10 \div \frac{2}{3}$
15. $12 \div \frac{3}{4}$
16. $15 \div \frac{4}{5}$
17. $16 \div \frac{3}{8}$
18. $14 \div \frac{2}{5}$
19. $20 \div \frac{4}{9}$
20. $25 \div \frac{5}{7}$

21. How many boxes each holding $\frac{1}{2}$ pound are required to hold 8 pounds of candy?

22. A drawing teacher has on hand 12 yards of narrow ribbon. How many pupils can she supply with $\frac{3}{4}$ yard each for their calendars?

23. Out of a class of 20 how many pupils must wait for a later supply, if each is to have $\frac{1}{2}$ yard of cord, and the drawing teacher has only 8 yards?

24. How many book ends each requiring $\frac{1}{2}$ square foot of tin may be cut from a sheet containing 6 square feet?

25. Allowing $\frac{1}{2}$ yard each, how many towels can be cut from a 7-yard piece of linen?

26. How many card cases, requiring $\frac{1}{4}$ square foot of leather, can be made from a pig skin, containing 4 square feet, not allowing for waste?

27. How many wrist purses, requiring $\frac{1}{12}$ square feet of leather, can be made from the same sized skin?

28. Allowing $\frac{1}{6}$ of a quart of cream apiece, how many children can be served with 3 quarts of ice cream?

29. Which will fill an exact number of sacks for a candy sale —to put 4 pounds of stuffed dates into bags holding $\frac{3}{4}$ pound or to put $\frac{2}{3}$ pound into each sack?

30. Betty sent to a publishing house $4.00 which her class had paid to subscribe to a weekly newspaper. If each subscription was $\$\frac{1}{5}$, how many copies of the paper were subscribed for?

Exercise 25. Dividing a Fraction by a Fraction

1. What is $\frac{7}{8} \div \frac{3}{5}$?

1st method. We can not divide 7 eighths by 3 fifths in the form in which they stand, so we first change them to fortieths.

$$\frac{7}{8} = \frac{35}{40} \text{ and } \frac{3}{5} = \frac{24}{40}.$$

$$35 \text{ fortieths} \div 24 \text{ fortieths} = \frac{35}{24} \text{ times, or } 1\frac{11}{24}.$$

2nd method. We found in Exercise 24 that $1 \div \frac{3}{5} = \frac{5}{3}$.

$\frac{7}{8} \div \frac{3}{5}$ will give $\frac{7}{8}$ as great a quotient as $1 \div \frac{3}{5}$.

Then $\frac{7}{8} \div \frac{3}{5} = \frac{7}{8} \times \frac{5}{3} = \frac{35}{24}$, or $1\frac{11}{24}$.

We see that the second method is much easier because it saves us the trouble of reducing both fractions to a common denominator.

In dividing one fraction by another use the following:

PRINCIPLE: To divide a number by a fraction, invert the divisor and multiply.

Divide:

1. $\frac{3}{8} \div \frac{3}{4}$
2. $\frac{2}{3} \div \frac{4}{5}$
3. $\frac{9}{12} \div \frac{3}{8}$
4. $\frac{3}{4} \div \frac{5}{8}$
5. $\frac{11}{14} \div \frac{3}{7}$
6. $\frac{2}{3} \div \frac{2}{9}$
7. $\frac{4}{5} \div \frac{5}{9}$
8. $\frac{15}{16} \div \frac{3}{4}$
9. $\frac{3}{4} \div \frac{2}{3}$
10. $\frac{7}{8} \div \frac{3}{4}$
11. $\frac{7}{12} \div \frac{3}{4}$
12. $\frac{3}{8} \div \frac{1}{2}$
13. $\frac{2}{3} \div \frac{1}{4}$
14. $\frac{3}{4} \div \frac{1}{2}$
15. $\frac{8}{9} \div \frac{4}{6}$
16. $\frac{12}{15} \div \frac{3}{7}$
17. $\frac{11}{12} \div \frac{2}{3}$
18. $\frac{1}{4} \div \frac{2}{3}$
19. $\frac{15}{19} \div \frac{5}{7}$
20. $\frac{15}{16} \div \frac{7}{8}$
21. $\frac{7}{8} \div \frac{5}{6}$
22. $\frac{3}{4} \div \frac{5}{9}$
23. $\frac{2}{7} \div \frac{4}{5}$
24. $\frac{3}{4} \div \frac{2}{7}$
25. $\frac{1}{9} \div \frac{3}{8}$
26. $\frac{1}{2} \div \frac{2}{3}$
27. $\frac{12}{42} \div \frac{4}{7}$
28. $\frac{15}{16} \div \frac{2}{3}$
29. $\frac{6}{7} \div \frac{2}{5}$
30. $\frac{7}{8} \div \frac{5}{8}$
31. $\frac{3}{4} \div \frac{2}{5}$
32. $\frac{11}{12} \div \frac{3}{10}$
33. $\frac{24}{25} \div \frac{2}{3}$
34. $\frac{6}{7} \div \frac{2}{9}$
35. $\frac{4}{5} \div \frac{7}{9}$
36. $\frac{18}{19} \div \frac{3}{4}$
37. $\frac{13}{14} \div \frac{2}{9}$
38. $\frac{9}{10} \div \frac{11}{12}$
39. $\frac{4}{9} \div \frac{29}{30}$
40. $\frac{14}{15} \div \frac{11}{12}$

41. Richard uses $\frac{1}{8}$ pound chocolate in making a certain fudge recipe. How many such recipes of fudge should he be able to fill from a $\frac{1}{2}$-pound cake of chocolate?

42. Three-fourths of a pound of shelled nuts are enough for how many small loaves of nut bread, each loaf requiring $\frac{1}{8}$ pound of nuts?

43. It took $\frac{1}{3}$ can of paint to finish each pair of tin book ends made in a certain handwork class. An opened can of paint $\frac{3}{4}$ full should finish how many book ends?

44. Painted flower pots required only $\frac{1}{4}$ can. This can $\frac{3}{4}$ full should be enough to finish how many flower pots?

45. A remnant of leather containing $\frac{5}{6}$ of a square foot should be enough for how many wrist purses requiring $\frac{1}{12}$ square foot? Allow $\frac{1}{10}$ square foot waste for careless cutting.

46. Albert wished to buy 15 apples for a party. He asked the storekeeper to weigh an apple. They found a medium sized one to weigh $\frac{1}{3}$ pound. How many pounds must he buy to obtain 15 apples?

Exercise 26. Dividing by a Mixed Number

As we noted in the multiplication of mixed numbers, it was much easier to reduce the mixed number to an improper fraction and then treat it as a fraction, so in dividing by a mixed number it is easier to reduce the mixed number to an improper fraction and follow the principles for the division by a fraction (see page 80).

Divide:

1. $9 \div 1\frac{3}{5}$
2. $6 \div 1\frac{3}{4}$
3. $9 \div 1\frac{1}{2}$
4. $16 \div 2\frac{2}{3}$
5. $18 \div 3\frac{3}{4}$
6. $7 \div 2\frac{1}{2}$
7. $15 \div 2\frac{2}{3}$
8. $3\frac{1}{2} \div 2\frac{5}{4}$
9. $2\frac{2}{3} \div 1\frac{3}{5}$
10. $10\frac{2}{3} \div 2\frac{2}{5}$
11. $4\frac{1}{2} \div 2\frac{4}{7}$
12. $7\frac{4}{5} \div 3\frac{1}{4}$
13. $5\frac{3}{4} \div 1\frac{7}{8}$
14. $3\frac{2}{5} \div 1\frac{8}{9}$
15. $16 \div 2\frac{2}{3}$
16. $18 \div 6\frac{3}{4}$
17. $6\frac{1}{2} \div 1\frac{4}{9}$
18. $3\frac{3}{4} \div 1\frac{3}{7}$
19. $5\frac{6}{7} \div 1\frac{5}{6}$
20. $16\frac{1}{2} \div 2\frac{3}{4}$
21. $17\frac{1}{2} \div 5\frac{1}{2}$
22. $36 \div 2\frac{1}{4}$
23. $19 \div 2\frac{1}{4}$
24. $\frac{7}{8} \div 1\frac{3}{4}$
25. $15\frac{1}{2} \div 3\frac{1}{3}$
26. $12 \div 2\frac{1}{2}$
27. $4\frac{1}{2} \div 1\frac{1}{2}$
28. $8\frac{2}{5} \div 4\frac{2}{3}$

Exercise 27. Practical Problems in Fractions

1. Ethel's mother has made $8\frac{1}{4}$ quarts of apple jelly and has asked Ethel to get ready enough $\frac{1}{2}$-pint glasses to hold it. How many were needed?

2. In order to get an exact number of rows in his garden, should John set out $\frac{2}{3}$ or $\frac{5}{6}$ dozen cabbage plants to a row, there being $7\frac{1}{2}$ dozen plants in all?

3. How many ferns at $\$\frac{1}{4}$ each can Dorothy buy for her fernery with $\$1\frac{1}{4}$?

4. If, during berry season, Carl earns on the average $\$1\frac{3}{4}$ a day, how many days must he work to earn $\$26\frac{3}{4}$ for a bicycle?

5. How many $\frac{1}{5}$-bushel baskets will $2\frac{1}{2}$ bushels of grapes fill?

6. How many tea towels, each requiring a $\frac{1}{2}$-yard length, may be cut from a remnant of toweling $3\frac{3}{4}$ yards long?

7. A remnant of "honeycomb" cloth $2\frac{1}{2}$ yards long may be made into how many dish cloths requiring $\frac{1}{3}$ yard each?

8. How many pans are needed to hold $3\frac{1}{2}$ dozen cookies at one baking, each pan holding $\frac{3}{4}$ dozen cookies?

9. How many $\frac{1}{8}$-pound sacks will $2\frac{1}{2}$ pounds of blanched almonds fill?

10. Into how many $\frac{2}{5}$-hour periods can a $5\frac{1}{5}$-hour school day be divided?

11. A group of girls together buy a 10-yard bolt of nainsook. How many aprons can they make from it if each apron requires $2\frac{1}{2}$ yards?

12. George and Helen found that they could make $4\frac{1}{2}$ quarts of lemonade for 25 cents. What was the cost of 1 quart?

13. Counting 4 glasses to a quart, find the cost of 1 glass.

14. How many holder covers $5\frac{1}{2}$ inches long can be cut from material a yard long and of the proper width? (1 yard = 36 inches.)

15. Marian and her mother, in dressing dolls for a bazaar, used a certain little dress pattern requiring $\frac{3}{8}$ yard of material. How many could they cut from a 2-yard remnant which Marian's mother had?

16. How many shelf boards $2\frac{2}{3}$ feet long can Richard cut from a 12-foot board of proper width?

17. How many stamped envelopes at $2\frac{1}{2}$ cents apiece can one buy for a quarter?

18. Herman, who lived in the country, walked $6\frac{2}{3}$ miles a day in order to attend a city high school. At this rate, how long was it before he had walked a hundred miles?

19. A train averaging $45\frac{1}{2}$ miles an hour should make a 500-mile run in how many hours?

20. A boy who, in a trial speed test, correctly works 2 multiplication problems in $2\frac{1}{2}$ minutes should, at the same rate, be able to solve how many of equal difficulty in a final 6-minute test?

21. How many aprons requiring $2\frac{3}{4}$ yards each can be made from $16\frac{1}{2}$ yards of cloth?

22. The smaller girls in a sewing class found their smocks required $2\frac{1}{2}$ yards apiece. How many could be cut from a $12\frac{1}{2}$-yard length of poplin?

23. Geraniums in bloom were selling at \$$1\frac{1}{4}$ a dozen. A class having \$$1\frac{3}{4}$ with which to buy plants for a window box could get how many at this price?

24. Jasper's garden is $7\frac{1}{2}$ yards long. Into how many plots $1\frac{1}{2}$ yards long can he divide it?

25. What is the average speed of a train which travels $93\frac{3}{4}$ miles in $2\frac{1}{2}$ hours?

26. How many times can $5\frac{1}{4}$ gallons of gasoline be drawn from a barrel that contains $31\frac{1}{2}$ gallons?

27. A certain kind of barbed wire weighs $1\frac{3}{8}$ pounds per rod. How much does it weigh per foot? ($16\frac{1}{2}$ feet = 1 rod.)

28. How many bushels of popcorn at \$$1\frac{1}{2}$ a bushel must be traded to pay for a sweater vest costing \$$4\frac{1}{2}$?

Exercise 28

Review Problems

A boy having a new pedometer said he intended to walk at least 4 miles every day. For the first week the distances registered by the pedometer in the evening of each day were as follows:

	Day	Daily register	Distance walked during the day	How much more or less than 4 miles?
1.	Monday......	$4\frac{2}{5}$ miles	?	?
2.	Tuesday......	$9\frac{3}{5}$ miles	?	?
3.	Wednesday...	$12\frac{3}{10}$ miles	?	?
4.	Thursday.....	$16\frac{3}{5}$ miles	?	?
5.	Friday.......	$20\frac{1}{2}$ miles	?	?
6.	Saturday.....	$24\frac{7}{10}$ miles	?	?

7. The second week he planned to walk at least 25 miles. The final reading on the second Saturday was $60\frac{2}{5}$ miles. How many miles did he walk during the week?

8. By how many miles had he exceeded his plan?

9. A sewing class cut the following lengths at different times from a 10-yard bolt of ribbon: $2\frac{1}{2}$ yards; $1\frac{3}{4}$ yards; $\frac{1}{2}$ yard, and $2\frac{3}{4}$ yards. How many yards were cut off in all?

10. How many of the original 10 yards remained?

11. After another girl used $1\frac{7}{8}$ yards, how long was the remnant that was left?

12. Three-eighths of a small farm is planted in apple orchards; $\frac{1}{4}$ in alfalfa; $\frac{1}{4}$ in canteloupes, and $\frac{1}{16}$ in small fruits and vegetables. What fraction of the whole farm is left for the house and yard?

13. In canning some fruit, a woman uses 10 pounds of sugar from a sack and $2\frac{3}{4}$ pounds from a bin. How many pounds did she use?

14. Twelve pounds of flour have been used from a sack whose net weight is $24\frac{1}{2}$ pounds. How many pounds are left?

15. When $5\frac{1}{2}$ more pounds of the flour have been used, how many pounds are left?

16. John weighed $62\frac{1}{2}$ pounds when school closed in the spring. During the summer he gained $5\frac{3}{4}$ pounds. How much did he weigh at the end of the summer?

17. A boy left the grocery store with packages in his basket, as follows: sugar, 5 pounds; grapenuts, $\frac{7}{8}$ pound; vanilla, $\frac{1}{8}$ pound; chocolate, $1\frac{1}{4}$ pounds; jar of olives, $1\frac{1}{4}$ pounds; tea, $\frac{1}{2}$ pound; loaf sugar, 2 pounds; can of molasses, $2\frac{3}{8}$ pounds; can of coffee, $\frac{5}{8}$ pound; bread, $1\frac{1}{4}$ pounds; box of dates, $\frac{7}{8}$ pound. The entire weight of his purchases with the basket, $\frac{7}{8}$ pound, was how many pounds?

18. Finding the load too heavy, he had his little brother take the bread, coffee and molasses. The younger boy was carrying how many pounds?

19. There were then how many pounds in the basket?

20. In a track meet William jumped $11\frac{3}{4}$ feet and Fred jumped $12\frac{1}{3}$ feet. How much farther did Fred jump?

21. Gilbert weighs $64\frac{1}{2}$ pounds and Mary weighs $61\frac{3}{4}$ pounds How much heavier is Gilbert?

Exercise 29. A Room Picnic

Applied Problems in Fractions

A fifth grade consisting of 32 children planned to have a room picnic.[1] They wished to plan a lunch that would not exceed 25 cents each. They appointed a committee to get prices on different things which they planned for the lunch. This committee brought the following price list to class:

1. For sandwiches		2. For salad	
Rolls, per dozen	$0.12	New potatoes, per peck	$0.40
Bread, per loaf	.10	Eggs, per dozen	.32
Butter, per pound	.36	Onions, per pound	.08
Cold boiled ham, per pound	.40	Celery, per bunch	.05
Peanut butter, per pound	.20	Green peppers, each	.05
Head lettuce, per head	.10	Dressing, per bottle	.15
Luncheon cheese, per jar	.15		

3. Meat		4. Lemonade	
Bacon, per pound	.32	Lemons, per dozen	.40
Frankfurts, per pound	.18	Sugar, per pound	.07½
Round steak, per pound	.26	Ice, 25 pound piece	.10

5. Ice Cream		6. Cake	
Any flavor, per quart	.30	Square angel food	.40
Cost of packing, per freezer	.25	Small loaf cakes	.10

The class first figured on the sandwiches. They measured a ten-cent loaf of bread and found that it measured 8½ inches long.

1. If the bread was cut ¼ of an inch thick, how many slices would be obtained from one loaf?

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[1] This exercise may be replaced by an actual situation in which the pupils figure up a lunch for their own class. If an outdoor picnic is not practical, an indoor picnic could be substituted.

2. By cutting the slices into two pieces, a sandwich could be made out of each slice. They agreed that 3 of these sandwiches should be provided for each person. Besides the 32 pupils, their teacher and one guest were to go on the picnic. How many loaves of bread were needed?

3. Find the cost of the bread.

They next decided to allow $1\frac{1}{2}$ rolls apiece for making "red hots."

4. How many rolls were needed for the whole party? How many dozen rolls should they buy?

5. Find the cost of the rolls.

Their next problem was to find how much butter would be needed. One girl reported that it took $\frac{1}{2}$ pound of butter for 4 dozen sandwiches of the size planned by the class.

6. How much butter was needed for the sandwiches and rolls at that rate?

7. Find the cost of the butter.

The class next voted on the sandwich filling. They decided to make the filling of lettuce and peanut butter.

8. They figured that they would need about $1\frac{1}{2}$ pounds of peanut butter and 6 heads of lettuce. Find the cost of these two items.

A boy counted the number of new potatoes in a peck that his mother had bought and found that there were 54.

9. Allowing $1\frac{1}{2}$ potatoes per person for salad, how many potatoes were needed for the whole group? This made approximately how many pecks?

10. Find the cost of the potatoes.

11. How much less was this than the amount set as the limit for each one's share?

12. The other materials for the salad were selected as follows: $\frac{3}{4}$ of a pound of onions; 3 bunches of celery; 2 green peppers; $\frac{3}{4}$ of a dozen eggs, and 2 bottles of salad dressing.

13. Figure the entire cost of the salad.

14. At this point, some were afraid there would be no money left for ice cream and cake. The teacher directed them to find the total cost so far as they had planned. What was the total?

15. What was each one's share so far?

A vote was taken on meat to cook over a fire. This vote resulted in a majority in favor of frankfurts. The butcher said that the size desired averaged about 12 to the pound.

16. Allowing 2 frankfurts apiece, how many pounds would be required for the whole group?

17. Find the cost of the frankfurts.

18. Allowing 1 lemon to 4 glasses of lemonade, how many lemons were needed for $1\frac{1}{2}$ glasses apiece? Besides the lemons they used 2 pounds of sugar and bought 10 cents worth of ice.

19. Find the cost of the lemonade.

In figuring on the ice cream, they decided to allow $\frac{1}{6}$ of a quart for each person.

20. How many quarts would they need at that rate for the whole group?

21. They decided to order 6 quarts. What part of a quart was this more than the total allowance found in problem 20?

22. Find the cost of the ice cream. Allow 25 cents for packing.

23. They also ordered 2 square angel-food cakes.

24. Find the total cost of the lunch for the group.

25. Find the amount required for each person.

26. How much did the amount per person, as they planned the lunch, differ from the original limit set for each person?

27. How many cents should be collected from each person? (The teacher asked that she be allowed to pay for the guest.) What would you have suggested doing with the few cents that would be left by having to collect in whole cents rather than in fractions?

Exercise 30. A Camp Fire Meal

Applied Problems in Fractions

These Boy Scouts are preparing a meal. They found this recipe for making corn bread on a camp fire and are trying it out:

$1\frac{1}{2}$ cups cornmeal
1 tablespoon salt
$\frac{3}{4}$ cup milk or water
2 eggs
1 tablespoon sugar
$1\frac{1}{2}$ cups flour
3 level teaspoonfuls baking powder
Butter the size of an egg (melted) or bacon drippings.

Mix dry ingredients together and stir in eggs and milk just before cooking. Pour batter into well greased hot skillet and cook over the coals. Enough for 4 hungry people.

1. There are 6 boys in this camping party. How much of each of the ingredients should they use to provide enough for six?

2. How much of each ingredient would be needed to make corn bread for 2 people?

Bacon, eggs and milk with corn bread form an appetizing out-of-doors meal. $1\frac{1}{2}$ pounds of bacon, $\frac{2}{3}$ of a dozen eggs and 2 quarts of milk are enough for 4 people.

3. Plan the amount of bacon, eggs and milk that would be needed by the 6 boys to go with their corn bread.

4. How much bacon, eggs and milk would be needed with the corn bread for 2 people?

5. How much of each article should be provided for a camping party of 8 for one meal?

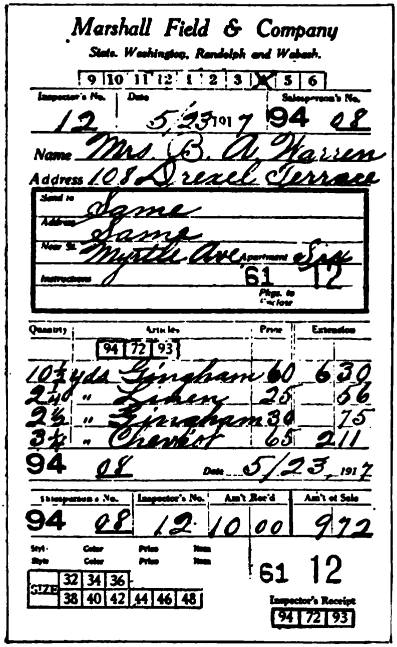
6. Find the approximate cost of this meal, using local prices, for each of the 6 boys in the camping party.

7. Plan another meal, using different provisions for such a camping party. Compare the cost with that in Problem 6.

Problems in Business

The following list of problems represents real business transactions. The items listed in each problem were copied from actual sales slips, supplied through the courtesy of a large and well-known mercantile house. You are here required to solve problems which had to be calculated by the salespeople in this great business house when making out the original sales slips and invoices.

Since these problems are to provide drill in fractions, the items on the regular sales slips not relating to fractions were omitted in copying.



Marshall Field & Company

State. Washington, Randolph and Wabash.

9 10 11 12 1 2 3 ~~4~~ 5 6

Inspector's No.	Date	Salesperson's No.
12	5/23 1917	94 08

Name Mrs B. A. Warren

Address 108 Drexel Terrace

Send to Same

Address Same

Near St. Myrtle Ave, Apartment Six

Instructions 61 12

Pkgs. to Enclose

Quantity	Articles	Price	Extension
	94 72 93		
$10\frac{1}{2}$	yds Gingham	60	6 30
$2\frac{1}{4}$	" Linen	25	56
$2\frac{1}{2}$	" Gingham	30	75
$3\frac{1}{4}$	" Cheviot	65	2 11

94 08 Date 5/23, 1917

Salesperson's No.	Inspector's No.	Am't Rec'd	Am't of Sale
94 08	12	10 00	9 72

Style Color Price Item

Style Color Price Item

61 12

SIZE	32	34	36			
	38	40	42	44	46	48

Inspector's Receipt

94 72 93

Exercise 31

Find the total amount for the purchases in each of the following problems:

1. Mrs. Warren bought $10\frac{1}{2}$ yards of gingham at 60 cents[1]; $2\frac{1}{4}$ yards of linen at 25 cents; $2\frac{1}{2}$ yards of gingham at 30 cents, and $3\frac{1}{4}$ yards of cheviot cloth at 65 cents.

2. Miss Goodwill ordered $\frac{1}{2}$ yard chambray at 60 cents; $1\frac{3}{4}$ yards linen at 90 cents, and $\frac{3}{4}$ yards linen at 85 cents.

3. Mrs. Van Dorn purchased $1\frac{1}{4}$ yards lace at $1.10; $1\frac{1}{2}$ yards lace at 65 cents; $4\frac{1}{6}$ yards spangled net at $10.00, and $\frac{5}{8}$ yard spangled net at $11.50.

4. Mrs. Campbell bought the following goods: $2\frac{1}{4}$ yards silk at $1.50; $2\frac{1}{3}$ yards silk at $1.00; $3\frac{1}{2}$ yards silk at $2.75, and $7\frac{1}{2}$ yards edging at 10 cents.

5. Mrs. Gregory purchased $14\frac{1}{2}$ yards silk at 95 cents; $1\frac{2}{3}$ yards fringe at $1.75; $5\frac{1}{2}$ yards sundour at $2.50; $2\frac{3}{4}$ yards fringe at 40 cents, and $1\frac{2}{3}$ yards denim at 35 cents.

6. Mrs. Dickson ordered $\frac{3}{4}$ yard lace at $2.75; $\frac{3}{4}$ yard tulle at $2.25, and $\frac{2}{3}$ yard net at $1.45.

7. Mrs. Johnson bought $1\frac{1}{2}$ yards of edging at 7 cents; $2\frac{1}{2}$ yards lace at 4 cents; $1\frac{1}{8}$ yards lace at 60 cents, and $1\frac{1}{2}$ yards lace at 25 cents.

8. Mrs. Herbert bought $5\frac{3}{4}$ yards cretonne at 30 cents; $2\frac{1}{2}$ yards burlap at 30 cents, and $1\frac{1}{2}$ yards denim at 35 cents.

9. Miss Young purchased $\frac{5}{8}$ yard of dress goods at 65 cents; $7\frac{1}{2}$ yards dress goods at 65 cents; $1\frac{3}{4}$ yards dress goods at $1.00, and $\frac{7}{8}$ yard veiling at 50 cents.

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[1] An expression such as $10\frac{1}{2}$ yards of gingham at 60 cents means $10\frac{1}{2}$ yards of gingham at 60 cents a yard. This form of statement will be used in all the problems. Any fraction less than $\frac{1}{2}$ cent should be discarded in entering the cost of any item on a sales slip and $\frac{1}{2}$ cent or more should be counted as another cent.

10. Mrs. Spaulding ordered the following: $2\frac{1}{4}$ yards trimming at 85 cents; $\frac{5}{8}$ yard trimming at \$3.00; $2\frac{1}{4}$ yards braid at 50 cents, and $1\frac{1}{2}$ yards cord at 8 cents.

11. Miss Jackson bought $16\frac{1}{4}$ yards of net at 50 cents; $3\frac{3}{4}$ yards muslin at 55 cents; $2\frac{1}{2}$ yards of dress goods at \$1.25, and $2\frac{1}{3}$ yards sundour at \$1.25.

12. Mrs. Adams purchased $4\frac{1}{2}$ yards black dress goods at \$3.00; $8\frac{3}{4}$ yards black dress goods at \$2.20; $3\frac{1}{2}$ yards velvet at \$1.25, and $4\frac{2}{3}$ yards braid at 50 cents.

13. Mrs. Howard bought $3\frac{1}{2}$ yards silk at \$1.25; $1\frac{1}{2}$ yards silk at 55 cents; $\frac{1}{2}$ yard silk messaline at \$1.25, and $\frac{2}{3}$ yard georgette crepe at \$1.95.

14. Miss Byron purchased $5\frac{1}{2}$ yards silk meteor at \$1.20; $\frac{3}{4}$ yard silk at \$1.25; $\frac{5}{8}$ yard silk net at \$1.75, and $1\frac{1}{4}$ yards silk at \$1.10.

15. Miss Loring ordered the following bill of goods: $1\frac{1}{2}$ yards ribbon at 28 cents; $\frac{1}{2}$ dozen handkerchiefs at \$1.50; $4\frac{1}{2}$ yards silk at \$1.00, and $4\frac{1}{2}$ yards braid at 15 cents.

16. Mrs. Gibson bought the following goods: $1\frac{1}{2}$ dozen handkerchiefs at \$1.20; $3\frac{1}{2}$ pairs No. 7642 curtains at \$7.00 per pair; $3\frac{1}{2}$ yards tapestry at \$1.35, and $9\frac{1}{2}$ yards scrim at 25 cents.

17. Miss Norris bought $2\frac{1}{2}$ yards filet net at 75 cents; $1\frac{3}{8}$ yards ribbon at 38 cents; $\frac{3}{4}$ yard leather at 75 cents, and $3\frac{1}{2}$ yards net at 55 cents.

18. Miss Howell bought the following: $4\frac{1}{2}$ yards marquisette at 30 cents; $4\frac{1}{2}$ yards edging at 5 cents, and $3\frac{3}{4}$ yards net at 40 cents.

19. Mrs. Drake ordered the following bill of goods: $30\frac{1}{4}$ yards cretonne at 25 cents; $9\frac{2}{3}$ feet pole at 4 cents per foot; $1\frac{1}{2}$ yards organdie at \$1.00, and $6\frac{1}{2}$ yards fringe at 7 cents.

20. Mrs. Chatfield purchased $4\frac{1}{2}$ yards net at 75 cents; $6\frac{2}{3}$ yards Swiss at 15 cents, and $\frac{5}{8}$ yard cretonne at 60 cents.

21. Mrs. Arnold purchased $1\frac{1}{3}$ yards dimity at 30 cents; $3\frac{1}{2}$ yards mull at 40 cents, and $10\frac{1}{2}$ yards cord at 8 cents.

22. Miss Penfield purchased $2\frac{1}{4}$ yards of braid at 10 cents and $4\frac{1}{4}$ yards cretonne at 60 cents.

23. Mrs. Madison bought $4\frac{1}{2}$ yards Jap toweling at 10 cents; $2\frac{1}{4}$ yards silkoline at 15 cents; $1\frac{1}{2}$ yards oil cloth at 25 cents, and $13\frac{1}{2}$ feet of rod at 5 cents per foot.

24. Miss Nelson purchased $1\frac{1}{2}$ yards Venetian lining at 50 cents; $4\frac{1}{2}$ yards satin at $1.00; $1\frac{1}{2}$ yards felt at 60 cents, and $3\frac{1}{2}$ yards satin at $1.35.

25. Miss Harding ordered $1\frac{3}{4}$ yards flouncing at $1.15; $2\frac{3}{4}$ yards lining at 50 cents; $1\frac{1}{2}$ yards lamb's wool at 85 cents, and $2\frac{1}{2}$ yards satin at 85 cents.

26. Mrs. Miller purchased the following: $1\frac{1}{2}$ dozen handkerchiefs at $1.20; $4\frac{1}{2}$ yards of net at 75 cents, $1\frac{1}{2}$ yards of ribbon at 28 cents and $9\frac{1}{2}$ yards of scrim at 25 cents.

27. Mrs. Dunn bought $5\frac{1}{4}$ yards of Jap toweling at 10 cents; $3\frac{1}{4}$ yards silkoline at 15 cents; $2\frac{1}{2}$ yards oilcloth at 20 cents, and 2 yards of silk at $2.00.

28. Mrs. Schaffer ordered the following bill: $6\frac{1}{2}$ yards fringe at 7 cents; $2\frac{1}{4}$ yards of braid at 10 cents; $4\frac{1}{4}$ yards cretonne at 60 cents, and $\frac{7}{12}$ of a dozen buttons at 40 cents.

29. Mrs. Martin purchased $1\frac{3}{4}$ yards of silk at 35 cents; $\frac{1}{4}$ yard silk brocade at $1.50; $\frac{3}{8}$ of a yard of taffeta at $1.75, and $\frac{1}{2}$ of a yard silk at 55 cents.

To insure correct calculations each charge is carefully checked before the final entry. You will readily see that, to be so efficient that his position is secure, a salesman must be thoroughly trained in speed and accuracy.

CHAPTER IV

Exercise 1. Decimal Fractions

You are already familiar with United States money. You have added, subtracted, multiplied and divided various sums of money.

1. How many dimes make a dollar?

2. How many cents make a dime? How many cents are there in a dollar?

3. How many mills make a cent? How many mills are there in a dollar?

4. Read this amount: $2.485.

The point which separates dollars from cents is known as the *decimal point*. The first figure (4) to the right of the decimal point in the above sum of money stands for 4 dimes.

5. Four dimes are equal to what part of a dollar?

The second figure (8) to the right of the decimal point in the above sum stands for 8 cents.

6. Eight cents are equal to what part of a dollar?

The third figure (5) to the right of the decimal point in that sum stands for 5 mills.

7. Five mills are equal to what part of a dollar?

8. Which is shorter to write: 2 dollars, 4 dimes, 8 cents and 5 mills or $2.485?

In the above questions we found that 4 dimes $= \frac{4}{10}$ of a dollar, 8 cents $= \frac{8}{100}$ of a dollar and 5 mills $= \frac{5}{1000}$ of a dollar. All of these fractions of a dollar have 10 or some multiple of 10 as their denominator.

A fraction whose denominator is ten or a product of tens (as 10; 100; 1000; 10,000 etc.) is called a decimal fraction.

We have already studied fractions in the form $\frac{4}{10}$, $\frac{8}{100}$ and $\frac{5}{1000}$ under the name of common fractions. A common fraction, however, may have any number for its denominator. Decimal fractions, on the other hand, can only have 10 or some multiple of 10 for their denominators.

The common fraction $\frac{4}{10}$ = the decimal fraction .4.

How, then, is a decimal fraction represented so that it appears different than a common fraction? This is done in United States money by using a decimal point and giving certain places to the right of the decimal point certain values.

We have already seen that 4 dimes (represented by the first place at the right of the decimal point) is $\frac{4}{10}$ of a dollar. In a decimal, then, the first place at the right of the decimal point is tenth's place.

.1 means one tenth.

The figure 8 in the second place at the right of the decimal point represents 8 cents or $\frac{8}{100}$ of a dollar. Then, the second place at the right of the decimal point is hundredth's place.

.01 means one hundredth.

The figure 5 in the third place at the right of the decimal point represents 5 mills or $\frac{5}{1000}$ of a dollar. Therefore, the third place at the right of the decimal point is thousandth's place.

.001 means one thousandth.

Which is shorter to write: $\frac{1}{1000}$ or .001?

Here we see that, as a matter of convenience in writing, the denominators of decimal fractions are not written as in common fractions, but the denominator is indicated by the number of places to the right of the decimal point.

Decimals were invented by the Scottish mathematician Napier (1550-1617). At first people were much prejudiced against them, but now everybody recognizes their convenience.

The other names applied to the various decimal places are shown in the following illustration:

hundred thousands	ten thousands	thousands	hundreds	tens	units	decimal point	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
1	1	1,	1	1	1	.	1	1	1,	1	1	1

This number is read: one hundred eleven thousand one hundred eleven *and* one hundred eleven thousand one hundred eleven *millionths.*

We see that the whole number is separated from the decimal fraction by the decimal point in writing and by the word *and* in reading. Also notice that the name of the last decimal place (millionths) is the name given to the decimal.

1. How does the 1 in tenth's place compare in value with the 1 in units' place?

2. How does the 1 in hundredth's place compare in value with the 1 in tenths' place?

3. Compare .2 with .02; .05 with .005; .001 with .1; .02 with .0002; 10 with .1.

4. Begin at the 1 in hundred-thousands place and go to the right. How do the values of the 1's change? Begin at the 1 in millionth's place and go the left. How do the values of the 1's change?

It will be a great convenience to you in reading decimals if you learn the number of places designated by each name. 1 place is read *tenths;* 2 places, *hundredths;* 3 places, *thousandths;* 4 places, *ten-thousandths;* 5 places, *hundred-thousandths;* and 6 places, *millionths.*

The word *and* should be used in reading numbers only between a whole number and a decimal.

Exercise 2

Read the following decimals:

1. .2; .02; .002
2. 1.2; 1.02; 1.002
3. 300.7; 8.012; 9.08
4. 0.005; 7.03; 1.03
5. .0005; 92.3; 75.02
6. .7854; 3.1416; 75.02
7. .32605; .000625; 7.9
8. 8.002; .00825; 3.014
9. .625; 60.06; 32.7
10. 1.015; 50.01; 2.0002
11. 100.001; .101; 100.101
12. 5.75; 19.002; 4.0067
13. 90.075; .866; 1.4142
14. 2150.42; 4.065; 3.003
15. 82.03; 92.057; 2.204
16. 2.54; .00125; .003175

Exercise 3

In writing decimals, ask yourself how many places there are in the decimal; then how many figures there are in the number you are to write and prefix the necessary number of zeros before the number to fill in the required number of decimal places.

For example: Write seventy-five ten-thousandths.

There are *four* places in the decimal; there are *two* figures in the number. Therefore, there must be two zeros prefixed to the 75 to fill in the required number of decimal places.

Then, seventy-five ten-thousandths =.0075.

In some books an extra zero is prefixed before the decimal point as here shown—0.075—in order that the decimal point stands out more clearly. It is omitted in most cases in this book, because it is regarded as unnecessary and requires extra time for writing it.

Write these decimals.

1. One one-hundredth.
2. Two hundred five ten-thousandths.
3. Sixty-three thousandths.
4. Twenty-five hundred-thousandths.
5. Seven hundred fifteen thousandths.
6. Eighty-seven thousandths.
7. Nine hundred forty thousandths.
8. Sixty-three hundredths.
9. Sixty-seven thousandths.
10. Eight millionths.
11. Three and one thousand four hundred sixteen ten-thousandths.
12. Five thousand two hundred thirty-six ten-thousandths.

Exercise 4. Addition of Decimals

In adding whole numbers, how do we arrange them?

Decimals are arranged in columns for addition in the same manner that whole numbers are, tenths being placed under tenths, hundredths under hundredths, etc. In what order will the decimal points fall?

Example: Add 10.7 and 21.92.

$$\begin{array}{r} 10.7 \\ \underline{21.92} \\ 32.62 \end{array}$$

Arrange these numbers so that the decimal points are in a vertical line. There are no hundredths to add to the 2 hundredths, so we bring the 2 hundredths down to the hundredths place in the sum. What is the sum of 9 tenths and 7 tenths? How many tenths make a whole unit? How many tenths are left out of 16 tenths after making a whole unit? Bring this number of tenths down in tenth's place in the sum. Add the 1 unit to the 1 unit in the units' column, making 2 units, and the 2 tens to the 1 ten, making 3 tens. The sum, then, is 32.62.

Add:

1. 2.7 and .3
2. 37.1 and 2.4
3. 105.6 and 50.4
4. 6.67 and .33
5. 9.99 and 0.01
6. 75.75 and 24.25
7. 73.25 and 1.75
8. 999.99 and 0.01
9. 28.8 and 1.27
10. 1000.01 and 999.09
11. 37.5 and 56.75
12. 3.12 and 31.2
13. 104.36 and 2.57
14. 28.75 and 71.25
15. 33.33$\frac{1}{3}$ and 66.66$\frac{2}{3}$
16. 28.48 and 71.12
17. .99 and .265
18. .009 and .991
19. 87.4 and 15.6
20. .0045 and 25.0055

21. Mrs. McFarland's light bills for the past six months were: $1.73; $1.68; $2.12; $1.88; $2.03, and $1.85. Find the total sum of her bills for the past six months.

22. Edwin bought the following bill of groceries at a store: sugar, $0.69; bread, $0.15; a can of salmon, $0.28; eggs, $0.32, and potatoes, $0.25. Find the amount of his bill of goods.

23. A Boy Scout fitted up a wireless station at the following cost for materials: 1 pound No. 20 copper wire, $0.46; 12 porcelain cleat insulators, $0.18; 1 pound No. 14 aluminum wire, $0.48; 1 double binding post, $0.12; 10 single binding posts, $1.00; old brass, $0.15; 2 feet $\frac{1}{4}$ inch brass rod, $0.20; tin foil, $0.10, and 1 pair of standard telephone receivers, $5.00. Find the total cost of his materials.

24. How many tons of coal were there in five loads weighing 1.42 tons, 1.382 tons, 1.463 tons, 1.35 tons and 2.481 tons?

25. Helen bought an arithmetic for $0.50, a reader for $0.45, a tablet for $0.05 and a bottle of library paste for $0.05. How much was the sum of her purchases?

26. The rainfall at a certain city in a recent year was as follows: January, 2.39 inches; February, 2.76 inches; March, 4.21 inches; April, 5.15 inches; May, 4.27 inches; June, 2.05 inches; July, 2.38 inches; August, 1.09 inches; September, 2.58 inches; October, 3.19 inches; November, 2.80 inches; December, 1.47 inches. What was the total rainfall for the year?

Exercise 5. Subtraction of Decimals

Decimals are subtracted in the same way that whole numbers are, the decimal points falling in a vertical line; tenths under tenths, hundredths under hundredths, etc.

Subtract 3.127 from 4.5.

$$\begin{array}{r} 4.500 \\ \underline{3.127} \\ 1.373 \end{array}$$

Arrange the two numbers with the decimal points in a vertical line. Then fill in the hundredth's and thousandth's places in the minuend with zeros. We can now subtract the two numbers as if they were whole numbers, putting in a decimal point in the remainder directly under the other decimal points.

Subtract the following:

1. 4.231 − 1.927
2. 6.5 − 3.25
3. 18.3 − 9.375
4. 24.85 − 16.39
5. 10 − 7.875
6. 4 − 1.003
7. 3.05 − 1.26
8. 800.75 − 775.25
9. 15.06 − 9.18
10. 396.09 − 247.9
11. 36.112 − 29.802
12. $45.25 − $29.35
13. $10 − $7.63
14. $25 − 19.07
15. 36 ft. − 6.3 ft.
16. 43.19 − 27.17
17. 2.125 − 1.0025
18. 6.5 − 3.625
19. $14.85 − $9.50
20. 716.88 − 528.4
21. 136.97 − 58.84

22. John had $48.75 in his savings bank account and took out $4.50 to buy some shoes. How much did he have left in his account?

23. Ruth made $12.00 during the vacation. She spent $5.85. How much did she have left?

Every year in different cities of our country there are held athletic contests at which championships in different sports are decided.

The ice skating records made at both indoor and outdoor contests in a recent year were as follows:

Event	U. S. Records	Canadian
220 yards	21.2 seconds	22.4 seconds
440 yards	41.2 seconds	40.2 seconds
½ mile	1.38 minutes	1.45 minutes
1 mile	2.92 minutes	3.106 minutes

24. Find which is the lowest record in each event, and how much—the Canadian or United States records.

25. A quart of milk weighs 2.155 pounds and a quart of water weighs 2.088 pounds. How much heavier is a quart of milk than a quart of water?

Exercise 6. Multiplication of Decimals

1. Multiply .25 by .5

Express .25 as a common fraction $\frac{25}{100}$ and .5 as a common fraction $\frac{5}{10}$. Then $\frac{25}{100}\times\frac{5}{10}=\frac{125}{1000}$.

$\frac{125}{1000}$ as a decimal $=.125$

Then $.25\times.5=.125$

How does the number of decimal places in the product (.125) compare with the number of decimal places in *both* the multiplier (.5) and the multiplicand (.25)?

2. Multiply .005 by .05

$$.005 = \frac{5}{1000} \qquad .05 = \frac{5}{100} \qquad \frac{5}{1000} \times \frac{5}{100} = \frac{25}{100000}$$

$$\frac{25}{100000} \text{ as a decimal } = .00025.$$

How does the number of decimal places in the product (.00025) compare with the number of decimal places in *both* the multiplier (.05) and the multiplicand (.005)?

3. Multiply 1.5 by 3.75

$$1.5 = 1\frac{5}{10} \text{ or } \frac{15}{10} \qquad 3.75 = 3\frac{75}{100} \text{ or } \frac{375}{100}$$

$$\frac{15}{10} \times \frac{375}{100} = \frac{5625}{1000} \text{ or } 5\frac{625}{1000}$$

$$5\frac{625}{1000} \text{ in the decimal form } = 5.625$$

How does the number of decimal places in the product (5.625) compare with the number of decimal places in *both* the multiplier (3.75) and the multiplicand (1.5)?

In the above examples we see the following[1]:

PRINCIPLE: In multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in both the multiplicand and the multiplier[1].

In multiplication of decimals it is not necessary to place units under units, tenths under tenths, etc.

4. Multiply .025 by 1.5

$$\begin{array}{r} .025 \\ \underline{1.5} \\ 125 \\ \underline{25} \\ .0375 \end{array}$$

Multiply as if the numbers were whole numbers. There are 3 decimal places in the multiplicand and one decimal place in the multiplier. Therefore, there must be 3+1, or 4 decimal places in the product. We must prefix a zero before the 3 and place the decimal point in front of this zero. Then .025 × 1.5 = .0375

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[1] To the teacher: All new principles should be developed inductively as shown above. If the pupils are not able to see the principle from the three illustrations that are given, additional examples can readily be supplied by the teacher until the pupils are able to see the principle involved.

Multiply the following:

5. .5×.27

6. 1.2×.75

7. .05×$175

8. .06×$142.50

9. .055×$275

10. 27.5×10.9

11. 2.5×79

12. 6×3.1416

13. 8×.866

14. 16×.7854

15. .0025×6.5

16. 396.09×1.5

17. 7.75×.075

18. .038×26.5

19. .48×1250.

20. 25.5×3.2

21. 79×.025

22. 12×3.1416

23. .06×355.25

24. .075×.062

25. .1×.001

26. .0525×875

27. 7.5×6.25

28. .075×4.25

29. 16.5×16.5

30. 5.5×5.5

31. 1.15×2.3

32. 7.5×3.1416

33. .375×4.8

34. 2150.42×3.5

35. 30.25×5.75

36. .06×$850.50

37. .05×$2000

38. .042×275 lb.

39. .038×156 lb.

40. .037×280 lb.

41. An owner of a factory employs 21 men at $2.50 per day, 14 men at $3.25 per day and 8 men at $3.75 per day. Find the total amount of wages paid per day.

42. A farm of 83.5 acres was sold for $122.75 per acre. Find the amount received for the farm.

43. Mr. Adams bought 8 tons of hard coal at $7.75 per ton. What was the cost of that amount of coal?

44. Philip went to visit his grandparents who lived 85 miles away. He traveled .3 of the distance by boat and the rest by train. How many miles did he travel each way?

45. A farmer had a drove of 64 hogs and sold .75 of them. How many did he sell?

46. A gardener sold a grocer 16 bushels of rutabagas at 85 cents a bushel. He received in exchange 125 pounds of sugar at $7\frac{1}{2}$ cents per pound, and the balance in cash. How much cash did he receive?

47. Find the cost of 1.75 cwt. of freight at $.24 per cwt. (Cwt. is an abbreviation of *hundred weight.*)

Exercise 7. Division of a Decimal by a Whole Number

1. Divide .8 by 4

Express .8 as the common fraction $\frac{8}{10}$

$\frac{8}{10} \div 4 = \frac{2}{10}$ $\frac{2}{10}$ expressed as a decimal = .2

2. Divide .08 by 4.

Express .08 as the common fraction $\frac{8}{100}$

$\frac{8}{100} \div 4 = \frac{2}{100}$ $\frac{2}{100}$ expressed as a decimal = .02

3. Divide .12 by 4

$.12 = \frac{12}{100}.$ $\frac{12}{100} \div 4 = \frac{3}{100}$ $\frac{3}{100}$ as a decimal = .03

4. Divide .025 by 5

$.025 = \frac{25}{1000}$ $\frac{25}{1000} \div 5 = \frac{5}{1000}$ $\frac{5}{1000} = .005$

We may now put these examples in the regular division form, pointing off the quotients by the answers obtained by using common fractions:

$$\begin{array}{r} .2 \\ 4\overline{).8} \end{array} \qquad \begin{array}{r} .02 \\ 4\overline{).08} \end{array} \qquad \begin{array}{r} .03 \\ 4\overline{).12} \end{array} \qquad \begin{array}{r} .005 \\ 5\overline{).025} \end{array}$$

PRINCIPLE: In dividing a decimal by a whole number the decimal point is placed in the quotient directly above the decimal point in the dividend.

Exercise 8. Division of a Decimal by a Decimal

1. Divide .025 by .05.

$.025 = \frac{25}{1000}$ $.05 = \frac{5}{100}$

$$\frac{25}{1000} \div \frac{5}{100} = \frac{\overset{5}{\cancel{25}}}{\underset{10}{\cancel{1000}}} \times \frac{\overset{1}{\cancel{100}}}{\underset{1}{\cancel{5}}} = \frac{5}{10}$$

$\frac{5}{10} = .5$ Therefore .025 ÷ .05 = .5

Other examples such as .5 ÷ .05; .04 ÷ .2; .8 ÷ .02; .05 ÷ .025; .75 ÷ .025, etc., should be worked out in a similar manner in connection with this example.

2. Divide 2.5 by .25

$$2.5=2\tfrac{5}{10}\text{ or }\tfrac{25}{10}\qquad .25=\tfrac{25}{100}$$

$$\frac{25}{10}\div\frac{25}{100}=\frac{\overset{1}{\cancel{25}}}{\cancel{10}}\times\frac{\overset{10}{\cancel{100}}}{\cancel{25}}=10.$$

Therefore $2.5\div .25=10$

3. Divide 9.375 by 37.5.

$$9.375=\tfrac{9375}{1000}\qquad 37.5=\tfrac{375}{10}$$

$$\frac{9375}{1000}\div\frac{375}{10}=\frac{\overset{25}{\cancel{9375}}}{\underset{100}{\cancel{1000}}}\times\frac{\overset{1}{\cancel{10}}}{\underset{1}{\cancel{375}}}=\frac{25}{100}.$$

$\frac{25}{100}=.25$. Therefore $9.375\div 37.5=.25$

We may now put these examples in the form of long division, so that the principle of division of decimals may be more clearly seen. The quotients are pointed off as obtained by using the common fractions above.

$$\begin{array}{r}
.5\\
.05\overline{).02\,5}\\
\underline{2\,5}
\end{array}
\qquad
\begin{array}{r}
10.\\
.25\overline{)2.50}\\
\underline{2\,5}\\
0
\end{array}
\qquad
\begin{array}{r}
.25\\
37.5\overline{)9.375}\\
\underline{7\,50}\\
1\,875\\
\underline{1\,875}
\end{array}$$

Answer the following questions for each of the above problems:

4. How many decimal places are there in the divisor?

5. How many decimal places to the right of the decimal point in the dividend is the decimal point in the quotient?

6. How does the number of decimal places to the right of the decimal in the dividend compare with the number of decimal places in the divisor?

PRINCIPLE: The decimal point in the quotient is as many places to the right of the decimal point in the dividend as there are decimal places in the divisor.

Exercise 9

1. Divide .032375 by .875.

$$\begin{array}{r} 0.037 \\ .875\overline{).032375} \\ \underline{2625} \\ 6125 \\ \underline{6125} \end{array}$$

There are 3 decimal places in the divisor. Count over three places to the right and place the decimal point in the quotient above the space at the right of the 2 in the dividend. After the decimal point has been located, the division is then performed as if the divisor and dividend were whole numbers.

2. Divide 45 by .15.

$$\begin{array}{r} 300. \\ .15\overline{)45.00} \end{array}$$

There are 2 decimal places in the divisor. We must add two zeros after the decimal point and place the decimal point in the quotient above the space at the right of the second zero in the dividend. Then divide as if the numbers were whole numbers.

3. Divide 11 by 15.

$$\begin{array}{r} .733+ \\ 15\overline{)11.000} \\ \underline{105} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array}$$

Since 11 is not exactly divisible by 15, we must add zeros for decimal places and carry out the division as a decimal. The decimal point in the quotient is placed directly above the decimal point in the dividend because there are no decimal places in the divisor. The sign + is used after the second 3 to indicate that there is still a remainder after carrying the quotient out to thousandths.

Divide the following:

(Carry out the quotients, if necessary, to at least 3 decimal places.)

4. $19 \div 1.9$

5. $1 \div .01$

6. $70 \div .7$

7. $45 \div .15$

8. $30 \div .75$

9. $1.2 \div .06$

10. $1.75 \div .25$

11. $45.5 \div 1.3$

12. $.704 \div 22$

13. $.072 \div .8$

14. $10 \div 3$

15. $10 \div 6$

16. 3÷8
17. 5÷8
18. 7÷8
19. 15÷24
20. 16÷64
21. 100÷33
22. 125÷.05
23. 9.5÷250
24. 689.85÷16425
25. 462.4÷7.4
26. 30.25÷5.5
27. 272.25÷16.5
28. .924÷2.64
29. 101÷.001
30. .0875÷2.5
31. 3.67692÷2.6
32. 10.392÷.866
33. 21.9912÷3.1416
34. $13.75÷$275
35. $43.50÷$725
36. $110÷$2000

In order to check the position of the decimal point in your result, it is a good plan to estimate each result approximately. For example: 1.3 is contained in 45.5 something over 30 times.

37. A grocer paid $7.80 for a case of 30 dozen eggs. How much did the eggs cost per dozen?

38. An automobile was driven 172 miles in 8 hours during a tour on the Lincoln Highway. Find the average speed in miles per hour.

39. A gallon of water weighs 8.355 pounds. How much does a quart of water weigh?

40. The expenses of a camping trip for a party of 14 Camp Fire girls were $37.94. If they shared the expenses equally, how much was each one's share?

41. Mr. Jenkins paid $68.85 for his winter's supply of 9 tons of hard coal. What was the cost per ton?

42. A farmer sold 5.5 pounds of butter to a grocer at $0.30 per pound, receiving in exchange sugar at $0.075 per pound. How many pounds of sugar did he receive?

43. In a motorcycle race the champion went 25 miles in 16.45 minutes. Find his speed in miles per minute.

44. In a roller skating tournament the winner skated 2 miles in 5.2 minutes. What was his average time per mile?

Exercise 10. Heights of Mountains

Photo, Underwood and Underwood N. Y.

How many feet are there in a mile? You will find this number in the first table on the inside of the back cover of this book. How far from the school house would you have to go to walk a mile? If you estimate a mile in this way, it will help you to picture the heights of the mountains given in the following table. Mt. Blanc, shown in the picture,[1] is slightly less than 3 miles high.

Find the height of the following mountains[2] in miles, carrying the result to at least three decimal places:

	Mountain	Continent	Height in Feet	Height in Miles
1.	Mt. Everest	Asia	29,002	?
2.	Aconcagua	South America	22,080	?
3.	Chimborazo	South America	20,498	?
4.	Mt. McKinley	North America	20,464	?
5.	Kilimanjaro	Africa	19,780	?
6.	Mt. Logan	North America	19,539	?
7.	Orizaba	North America	18,314	?
8.	Mt. Blanc	Europe	15,781	2.9509+
9.	Pike's Peak	North America	14,111	?
10.	Fujiyama	Asia	12,365	?
11.	Mt. Etna	Europe	10,835	?
12.	Kusciusko	Australia	7,836	?

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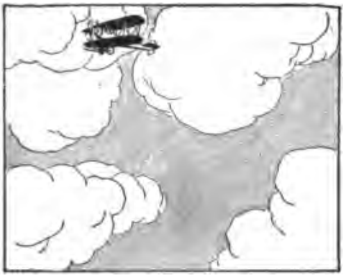
[1] This photograph shows the first passage of the Alps by aeroplane. This marvelous feat was accomplished by the French aviator Parmilin.

[2] The heights of most of these mountains are found on page 188 of the Efficiency Primary Arithmetic. At that time only the approximate height was found. Estimating the approximate height in miles is a good exercise to use here to precede finding the exact heights by carrying the result to at least three decimal places.

Exercise 11. Aeroplane Records



1. The record for height ascended in an aeroplane is held by Heinrich Oelreich who reached a height of approximately 24,606 feet. How many miles high did he go? Express the fractional part as a decimal, carrying the result to 3 decimal places.

2. How much higher is the summit of Mt. Everest than the height to which Oelreich ascended in his aeroplane?

3. The best American record for height was made by Captain H. L. Muller of the United States Army. He ascended 17,441 feet. Express this height in miles and a decimal of a mile.

4. How many miles higher was the record of Oelreich than that made by Captain Muller?

5. Victor Carlstrom, in an aeroplane, ascended 16,500 feet, carrying a passenger, April 30, 1916. How many miles high did the two ascend? What decimal fraction of a mile higher is this than the summit of Mt. Blanc.

6. H. G. Hawker, of Brooklands, England, made a record of 24,408 feet on April 26, 1916. What decimal fraction of a mile is his record below that of Oelreich?

7. The record for height in a gas balloon is 28,750 feet, made by Professor Berson in 1894. What decimal fraction of a mile higher is this than the record for an aeroplane?

8. The record for a dirigible balloon is 9,514 feet. It was made by the Clement-Bayard III, May 20, 1912. The dirigible carried six passengers. Express this record in miles, carrying the result out to 3 decimal places.

Exercise 12

1. Multiply 42.5 by 10; 3.75 by 10, and .625 by 10.

How has the position of the decimal point been changed by multiplying each number by 10?

2. Divide 42.5 by 10; 3.75 by 10, and .625 by 10.

How has the position of the decimal point been changed by dividing by 10?

From these examples, we see that

> **Multiplying a number by 10 moves the decimal point one place to the right, and dividing a number by 10 moves it one place to the left.**

3. How will the position of the decimal point be changed by multiplying or dividing a number by 100?

4. How will the decimal point be changed in position by multiplying or dividing a number by 1000?

Give the results of the following orally:

5. 62.5×10
6. 62.5÷100
7. 3.1416×100
8. 345.8÷1000
9. 62.5÷10
10. .866×10
11. 2150.42×100
12. 39.37÷100
13. .0075×100
14. 2.204×10
15. 272.25÷100
16. 6235÷1000
17. $1.75×100
18. 13.75÷10
19. 16.5÷100
20. $9.50×1000
21. $2.75×10
22. .0025×100
23. .625÷100
24. 87.75÷10
25. 8500÷1000
26. 153÷100
27. 3.1416×10
28. 1.4142×100
29. 87.5÷100
30. .0875×1000
31. 1465÷100
32. .055×1000
33. .866×100
34. 37.5÷100
35. .038×100
36. $.75×10
37. .5236×1000

38. How much will 10 dozen eggs cost at $0.28 per dozen?

Exercise 13. Changing Common Fractions to Decimals

A common fraction may represent an indicated division as well as a number of the equal parts of a thing. For example, $\frac{3}{8}$ may be read 3 divided by 8.

$3 \div 8 = .375$. Then, the common fraction $\frac{3}{8} = .375$.

To change a common fraction to a decimal, we divide the numerator by the denominator.

Change the following fractions to decimals:

1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. $\frac{2}{4}$
4. $\frac{3}{4}$
5. $\frac{1}{5}$
6. $\frac{2}{5}$
7. $\frac{3}{5}$
8. $\frac{4}{5}$
9. $\frac{1}{8}$
10. $\frac{5}{8}$
11. $\frac{7}{8}$
12. $\frac{1}{3}$
13. $\frac{2}{3}$
14. $\frac{1}{16}$
15. $\frac{1}{7}$
16. $\frac{24}{25}$

Exercise 14. Changing Decimals to Common Fractions

1. Change .375 to a common fraction.

$.375 = \frac{375}{1000}$. $\frac{375}{1000}$ reduced to its lowest terms $= \frac{3}{8}$.

2. Change $.33\frac{1}{3}$ to a common fraction.

$.33\frac{1}{3} = \frac{33\frac{1}{3}}{100}$. $\frac{33\frac{1}{3}}{100} = 33\frac{1}{3} \div 100 = \frac{\overset{1}{\cancel{100}}}{3} \times \frac{1}{\underset{1}{\cancel{100}}} = \frac{1}{3}$.

To change a decimal to a common fraction, write the decimal with its denominator indicated as in a common fraction and reduce the result to its lowest terms.

Change the following decimals to common fractions:

3. .25
4. .025
5. .8
6. .75
7. .6
8. .125
9. .4
10. .5
11. .875
12. .05
13. .625
14. $.66\frac{2}{3}$
15. $.14\frac{2}{7}$
16. $.16\frac{2}{3}$
17. $.11\frac{1}{9}$
18. $.06\frac{2}{3}$

19. Which is shorter, to take .375×64 or $\frac{3}{8}$ of 64?

20. Which is shorter, to find $.33\frac{1}{3}$×\$72.09 or $\frac{1}{3}$ of \$72.09?

Exercise 15. School Projects

Applied Problems in Fractions

In a certain school the children raise bulbs in connection with their nature study. One year the fifth grade did all the ordering and selling of the bulbs as a part of their work in arithmetic.

1. Find the amount of their first order for stock:

400 Crocuses, mixed	$2.25
100 Red tulips	1.00
100 Red and yellow tulips	1.50
100 Pink tulips	1.00
100 Yellow tulips	1.00
100 Blue hyacinths	2.75
100 Rose hyacinths	2.75
100 Narcissus poeticus	.90
200 Narcissus, paper white	3.50
100 Daffodils	2.00
12 Chinese lilies	.78
Total	?

2. Find the cost price of one bulb in each of the eleven kinds of bulbs. For example: 400 crocus bulbs cost $2.25. One bulb costs $2.25 ÷ 400, or $\frac{225}{400}$ of a cent, which equals $\frac{9}{16}$ of a cent.

The class then decided on the following price list and sent it to each room:

Crocuses, 4 for 3 cents
Red tulips, 1 cent each
Red and yellow tulips, 2 for 3 cents
Pink tulips, 1 cent each
Yellow tulips, 1 cent each
Hyacinths, 3 cents each
Narcissus poeticus, 1 cent each
Narcissus, paper white, 2 cents each
Daffodils, 2 cents each
Chinese lilies, 7 cents each

3. Find the gain on each kind of bulbs, providing the class sold all of the bulbs.

4. Find the total gain. This gain was to pay express charges and other incidental expenses. The express charges were $1.00. How much was left to pay the incidental expenses for sacks, money order to pay for the bulbs, etc.?

Among the problems which the clerks had to solve in the "bulb business" were the following:

Exercise 16

1. John pays for a Chinese lily and 6 red tulips with a quarter. Count his change.[1]

2. Elizabeth gave a half-dollar in payment for 2 dozen crocuses and 9 hyacinths. How much change should she have received?

3. Marion buys 6 "paper whites" and 3 Chinese lilies. She gives the cashier two quarters. Count her change.

4. Alexander bought 4 red and yellow tulips and 2 hyacinths. He gave the cashier 2 dimes. Count his change.

Exercise 17. Counting Money

1. At the end of one day there were in the cash box 1 quarter; 3 dimes; 8 nickles, and 3 pennies. Find the amount of the day's sales.

2. Another day the cashier reported a dollar bill, 1 fifty-cent piece; 2 quarters; 2 dimes; 9 nickels, and 5 pennies. Find the amount of that day's sales.

3. One noon the class sent the following money to the bank: two dollar bills; 3 fifty-cent pieces; 2 quarters; 2 dimes; 9 nickels, and 5 pennies. How much money was deposited?

---

[1] See note on the "Austrian Method," page 164 of the Primary Arithmetic. Follow this method here.

Making Sales Slips

Sold to 10–1–1916.
JOSEPH DUNLAP

3	Red Tulips @ 1 cent.	$0.03
2	Daffodils @ 2 cents..	.04
1	Chinese Lily........	.07
		$0.14
	D. K.	

The clerks made out sales slips which they gave to each purchaser to present to the cashier when he paid for his bulbs.

Cut strips of paper 3 inches by 4 inches and rule for sales slips as shown in the form on this page.

Make out slips for the following sales which were among those made the first two days. You may sell to any of your friends or classmates and sign your initials as if you were the clerk.

Exercise 18

1. 4 paper white narcissus; 1 Chinese lily; 3 red tulips; 1 rose hyacinth.

2. 8 "paper whites"; 2 rose hyacinths.

3. 1 Chinese lily; 4 black hyacinths; 6 red and yellow tulips; 8 "paper whites."

4. 8 crocuses; 4 red hyacinths; 2 "paper whites"; 5 daffodils.

5. 3 Chinese lilies; 4 yellow tulips.

6. 2 red and yellow tulips; 1 black hyacinth; 9 crocuses; 6 daffodils.

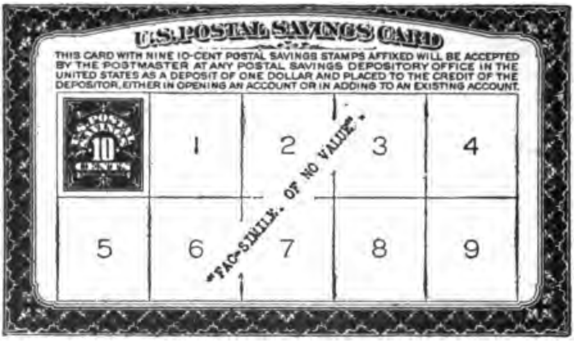
7. 12 pink tulips; 12 crocuses.

8. 1 Chinese lily; 1 "paper white"; 1 yellow tulip.

CHAPTER V

THRIFT

U. S. Postal Savings Banks



U.S. POSTAL SAVINGS CARD

THIS CARD WITH NINE 10-CENT POSTAL SAVINGS STAMPS AFFIXED WILL BE ACCEPTED BY THE POSTMASTER AT ANY POSTAL SAVINGS DEPOSITORY OFFICE IN THE UNITED STATES AS A DEPOSIT OF ONE DOLLAR AND PLACED TO THE CREDIT OF THE DEPOSITOR, EITHER IN OPENING AN ACCOUNT OR IN ADDING TO AN EXISTING ACCOUNT.

U.S. POSTAL SAVINGS
10
CENTS
1
2
3
4
5
6
7
8
9
FAC-SIMILE. OF NO VALUE.

To assist school children and others in opening savings accounts, the United States government issues a *Postal Savings Card* arranged as shown above.

An account can be opened with the government when the card is filled with stamps.

The purchase price of the card, 10 cents, and the cost of the 9 ten-cent stamps makes a total of $1.00 required to open an account in the postal savings bank.

Exercise 1

Solve as many of these problems orally as possible:

1. In Betty's room at school, after the advantages had been explained by the teacher, fourteen of the children purchased cards to open Postal Savings Accounts. Betty had previously saved 60 cents and her mother gave her 20 cents more. How many dimes did she need to complete her first dollar card?

2. A person joining the class which pays $1.00 a week should receive a check for how much at the end of the fifty weeks?

3. Betty's brother Glenn sold, at 65 cents each, four chickens which he had raised, and with the proceeds opened a Postal Savings Account. How much was his first deposit?

4. Glenn saved 70 cents from his earnings during that same week, which he deposited. How much had he then in the Postal Savings Bank?

5. Betty's cousin Robert had 50 cents and her cousin Myrtle had 40 cents which they invested in their first cards. How much additional did each need to open an account?

6. What was the total amount these four children had to their credit in Postal Savings at the end of their first week?

7. What was the average amount saved by each of the four children during this one week?

8. How much would the savings amount to for fourteen children at that rate?

9. If all of the 37 children in Betty's room had opened a savings account with an average of $1.10, how much would have been the total amount?

10. Frank earned 10 cents doing an errand, he found 10 cents in an old pocket, and his aunt and uncle each gave him 20 cents to help in paying for a Postal Savings Card like the one on the preceding page. How much did he still need to open a savings account with the government?

11. Frank saved 20 cents the following week. With that how many stamps did he still need to open his account?

12. Frank saved 30 cents during the third week. After completing his first card, how much had he then, with which to start another $1.00 card?

13. If each of these 37 children were able to make an average saving of 5 cents per week for 42 weeks of the year, how much would each have to his credit at the end of the year?

14. How much would the total amount be for all of the 37 children for the 42 weeks?

15. If the amount saved by each of the 37 children averaged 10 cents for each of the 42 weeks, how much would the total credit be?

16. Madeline put $10.95 in her Postal Savings Account during the first year. How many cents per day did she save on the average each day to equal that amount in the 365 days?

Christmas Savings Clubs

The Christmas Savings Club is an easy way of saving money which, in many places, has become popular among boys and girls as well as older people. Perhaps you have been a member of such a club. The plans are much the same in all banks. In some, the club is formed the middle of December. Each person who joins puts into the bank a certain amount each week for fifty weeks. Then, two weeks before Christmas he receives a check for all he has paid in.

Exercise 2

Try to get folders at local banks describing plans for Christmas and other savings clubs.

1. Mildred paid $0.25 a week for 50 weeks. How much did she receive two weeks before Christmas?

2. Her brother, two years older, saved $0.50 a week for the same length of time. How much did he receive from the Savings Club?

3. Richard paid 1 cent the first week, 2 cents the second week, 3 cents the third, and so on, increasing 1 cent a week for 50 weeks. Figure what he put in each week and find how much he saved altogether.

Put into 5 or more columns for adding instead of 1 column 50 numbers long.

4. In order to save time, Richard made his deposits once every five weeks in advance. Figure what he paid each time.

5. A boy who joins the class, paying 2 cents the first week, 4 cents the second, and increasing 2 cents each week for fifty weeks, receives how many times as much as Richard does? What is this amount?

6. Another class provides that the members pay 5 cents the first week, 10 cents the second, and increase 5 cents each week. Figure what must be paid each week, and find the total amount paid in.

7. In one class the members start with $2.50 the first week, $2.45 the second week, decreasing 5 cents each week. Find what members of this class receive at the end of the fifty weeks.

8. Lucy saved 1 cent each day to add to her savings account. How much did she save during the year?

9. Frank's father gave him $10 for his tenth birthday and increased his gift $1 each year afterwards. How much had Frank received when he was 21 years old?

10. How much is saved in a year of 50 weeks in a Christmas Club at 25 cents a week?

11. Helen joined a Christmas club which paid 10 cents a week for 50 weeks. She saved $7.65 in addition to her club account. How much did she save during the year?

School-Home Projects

Not only is it necessary to know some of the best plans for saving money, but it is equally important to know how to *earn* money. In a certain county nine-tenths of the schools have organized "School-Home Projects" for the purpose of earning and saving money.

Among the various projects in which the pupils are engaged are poultry raising, gardening, sewing, cooking and canning. The following problems are taken from the accounts which the pupils kept of their receipts and expenses.

Exercise 3

1. Albert received $110.86 from his poultry. The feed and other expenses amounted to $27.24. What were his net profits?

2. Elizabeth cultivated 1 acre of corn. Her receipts amounted to $45.35 and her net profits to $34.35. Find her expenses.

3. Elizabeth sold her corn for $.85 per bushel. How many bushels did she raise?

4. Mabel cleared $12.10 on the vegetables which she raised on a garden plot of 5½ square rods. What was her profit per square rod?

5. Elsie received $5.29 from a garden plot of 5 square rods of beets and onions. Her expenses were $1.75. What were her net profits?

In schools where "School-Home Projects" are not carried on, teachers should encourage pupils to keep accounts of home enterprises and make problems from them.

6. Willard purchased 3 geese for $6.00. The geese laid 30 eggs and hatched 25 goslings. Of this number, 12 did not live. He sold the remainder in the fall for $2.00 each. How much did he receive for the young geese?

7. Willard's feed bill in addition to what he himself supplied, was 50 cents. He still had the old geese. What were his net profits on the geese?

8. Ida bought 15 hens and made a profit of $7.85 from eggs. What was the average profit for each hen?

9. Harry had a plot of potatoes in the summer of 1916. His expenses were $1.85. He raised 24 bushels of potatoes and sold them at $1.20 per bushel. What were his profits?

10. Hilda cultivated 1.2 square rods of flowers. Her receipts were $14.95 and her expenses only 27 cents for seed. Find her net profits.

11. Find Hilda's net profit per square rod. How much would she have made on $\frac{1}{10}$ of an acre (16 square rods) at the same rate?

12. Harold planted a plot of 46.5 square rods of onion sets. His crop was 103.5 bushels, which he sold for 90 cents per bushel. What were his receipts for the onion sets?

13. Louis paid 75 cents to have his plot of land plowed. His other expenses were 38 cents for seed corn and 50 cents for planting. He raised 50 bushels of corn which he sold at 75 cents a bushel. How much did he clear on his farm project?

14. Walter and his sister, Viola, raised $\frac{3}{8}$ of an acre of potatoes. They sold them for 60 cents a bushel and received $27 for their entire crop. How many bushels did they raise?

15. William raised chickens one year. He sold $18.00 worth of eggs and $8.00 worth of young chickens. His feed cost him $7.75. How much profit did he make on the chickens?

16. Robert raised 5 bushels of popcorn which he sold to a grocer for $1.50 a bushel. The grocer re-sold the corn at 5 cents a pound. Had Robert sold the corn direct to the customers at that price, how much greater would have been his receipts? (A bushel of popcorn on the cob weighs 70 pounds.)

17. At a total cost of $6.20, Arthur purchased from pupils in his district a load of vegetables which he sold in the city as follows: radishes, 75 cents; lettuce, 65 cents; spinnach, 50 cents; beans, $2.00; peppers, $1.00; tomatoes, $1.50; beets, 90 cents, and corn, 60 cents. What profit did he make on the load?

18. A housekeeper purchased from Ruth 126 pounds of string beans at 5 cents a pound and canned them in 70 quart cans. What did each quart cost her?

19. What would have been Ruth's gain had she canned her own goods in pint cans costing $2\frac{1}{2}$ cents each and sold them at the market price of 10 cents a pound?

20. Mabel had $\frac{1}{8}$ of an acre planted in onions. She sold them as follows:

August 31, 6 bags at 90 cents each
October 4, 10 bags at $1.60 each
November 10, 15 bags at $2.00 each
November 27, 5 bags at $2.20 each

What were her total receipts? How many bags did she sell? What was the average price she received per bag?

21. Emma bought 30 cents worth of yarn and took charge of the family darning for the year. The family made the following purchases: 6 pairs of woolen hose at 75 cents each; 6 pairs of cotton hose at 25 cents each, and 9 pairs of lisle hose at 3 for $1.00. How much did her work add to the family saving, if darning doubles the life of a stocking?

22. There were 14 girls in a sewing club. They purchased the necessary material for their dresses together. The bill for the total amount was as follows:

35 yards gingham at 18 cents per yard
7 yards plain gingham for trimming at 25 cents per yard
14 cards buttons at 6 cents each
3 patterns at 15 cents each
7 spools white thread at 5 cents each
7 bolts seam tape at 10 cents each

Find the total cost. Find the cost for each dress.

23. Twelve pupils in one school reported the following receipts from the sale of their garden products: $58.60; $29.25; $72.00; $21.60; $27.21; $5.00; $1.61; $12.55; $15.82; $2.75; $50.00, and $4.89. Find the total amount earned by the pupils of this school.

24. What was the average amount earned by each pupil? If 15,000,000 pupils in the United States averaged that amount, how much would be the earnings from this source alone?

25. Evelyn made an apron in a sewing class. She bought 2 yards of gingham at 15 cents a yard, a card of braid for 5 cents, and a spool of thread for 5 cents. She sold her apron for 60 cents. How much did she gain?

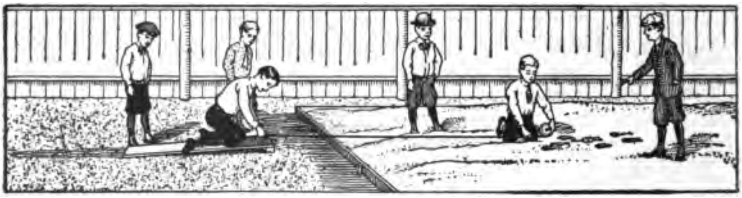
26. Meta made an apron, costing as follows: 1½ yards of gingham at 12 cents a yard; braid 5 cents; thread 5 cents. She sold her apron for 50 cents. What was her profit?

27. Which girl made the greater profit, Evelyn or Meta? How much more?

28. Wallace worked 2½ hours Monday, 3⅓ hours Tuesday, 4¾ hours Wednesday, 3¾ hours Thursday, and 2¼ hours Friday. He received 15 cents an hour for his work How much did his wages amount to in the five days?

CHAPTER VI

WEIGHTS AND MEASURES



Measuring the longest jump

Exercise 1. Measures of Length

1. If you were asked to find the length of your desk, what sort of measure would you use?

2. Measure the length and width of the top of your desk. How long is it? How wide is it?

3. When you go to a dry goods store to buy dress goods or ribbon, what measure does the clerk use? The clerk states the price of the dress goods or the ribbon at a certain number of cents or dollars a ———?

4. What unit of measure is usually used to express the width of a piece of ribbon or dress goods?

5. What would be the most convenient measure to use for measuring the length or width of your school room? Measure the length and width as suggested by the answers to this question.

If two or more measures are suggested, try both and decide which is the most convenient measure for that purpose.

6. Two boys wish to measure off the exact distance for a mile race. What measure should they use in order to measure it quickly and accurately?

7. Suppose that the boys were able to find only a yard stick and a ball of wrapping twine. Tell how they could make a more convenient measure than a yard stick for measuring such a long distance.

You have already seen the necessity for having different measures for measuring different kinds of distances. In order that we may be able to compare distances expressed in different units of measure, we need to know how many units of one kind make another kind of unit.

A girl in describing her garden plot said it was 33 feet long and 20 feet wide. A boy in the same class stated the length of his garden as 10 yards and the width as 8 yards.

8. Which plot was the longer?

9. Which plot was the wider?

You have already used most of the measures of length. You will find it convenient now to review these measures[1] of length by putting them in the form of a table.

12 inches[2] (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile

10. Draw a straight line on your tablet, freehand, as nearly 4 inches long as you can estimate it. Measure it with a ruler and see how much too long or too short it is.

---

[1] In olden times the length of an inch was found by placing three grains of barley lengthwise in a line. A certain king declared that his foot should be the unit for measuring length. The length of the yard is also supposed to be the length of the arm of one of the old English kings. In modern times, we need more exact measurements and all of our rulers, tapelines, carpenter's squares, etc., are supposed to agree with standard measures of length which are kept in the government buildings at Washington.

[2] Carpenters, architects and machinists use the sign ″ as an abbreviation for inches and the sign ′ for feet. For example, 3′ 6″ means 3 feet 6 inches.

11. Draw other lines[1] 2 inches; $3\frac{1}{2}$ inches; 6 inches; 5 inches, and 7 inches in the same way and check each by comparison with the ruler.

12. How many inches are there in a yard?

13. Draw a line, freehand, a yard long on the blackboard. Measure with a ruler or a yard stick to see how near a yard it is.

14. Stand, as nearly as you can estimate, a rod from the front wall of the school room. Have some other pupil measure this distance to see how closely you estimated that distance.

15. How many rods wide is the street or road in front of your schoolhouse? Measure this distance in rods to see how correctly you have estimated it.

Exercise 2

1. How many rods are there in a mile? How many feet are there in a rod? Then how many feet are there in a mile?

2. How many yards are there in a rod? How many rods are there in a mile? Then how many yards are there in a mile?

3. Find the number of inches in a rod.

4. Find the number of inches in a mile.

5. Change the following to inches: $1\frac{2}{3}$ feet; $2\frac{1}{2}$ feet; 3 feet 6 inches; 1 foot 8 inches; $1\frac{1}{2}$ yards; 1 yard 4 inches.

6. Change the following to feet: $1\frac{1}{3}$ yards; $2\frac{1}{2}$ yards; $1\frac{1}{4}$ rods; $\frac{1}{4}$ mile; $\frac{1}{2}$ mile; 84 inches; 5 rods; 144 inches.

7. Change the following to yards: 6 rods; 10 rods; 16 feet; 30 feet; 72 inches; $\frac{1}{4}$ mile; $\frac{3}{4}$ mile; $\frac{1}{2}$ mile; 120 feet.

---

[1] Make cards of various sizes and have pupils estimate the length and width of each as it is held before the class. This gives valuable training in estimating small distances.

8. In a certain city 10 blocks make a mile. How long is each block?

Distances on land are measured very accurately by surveyors. They use a surveyor's chain that is made of 100 links.

9. A surveyor's chain is 66 feet long. Find the length of each link in inches.

10. Caesar's Roman soldiers counted 2000 steps as a mile. If their mile had been exactly the same as ours how long was the average step of these soldiers?

11. In the American army, the marching step for quick time is $2\frac{1}{2}$ feet. How many such steps are there in a mile?

12. Walk 20 steps at your natural pace. Measure the distance that you have stepped and find the average length of each step. Some people become very expert in estimating distances by stepping them off. Practice stepping off and estimating distances.[1]

13. The Eiffel Tower in Paris is 984 feet in height. How many yards high is it? Find its height, expressed as a decimal fraction of a mile.

14. The span of the Brooklyn bridge is $1595\frac{1}{2}$ feet. What decimal fraction of a mile is the length of this span?

15. Railroads have their tracks divided into divisions called "blocks." Where the block system prevails no train can enter one of these divisions until the train ahead has left that "block." In a railway distance of 320 miles there are 80 equal "blocks." How long is each of these blocks?

16. Two of the races in a regular track meet are the quarter-mile and half-mile runs. How many yards are there in each of these distances?

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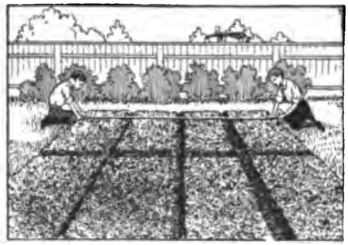
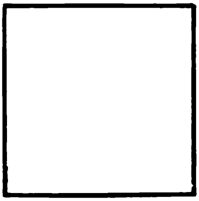
[1] The teacher should use local situations for his work.

Exercise 3. Square Measure



George and Harold are measuring Harold's garden plot. They find it to be 49.5 feet long and 33 feet wide. What units of measure should they use in describing the area or size of this garden plot?

The areas or sizes of such surfaces are usually expressed as equal to a certain number of square units.

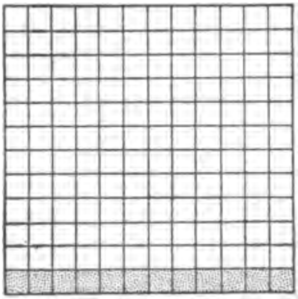


This figure is a square. How do its four sides compare in length? Since each side of this square is an inch in length, the square is called a *square inch.*

1. How long is the side of a square foot?

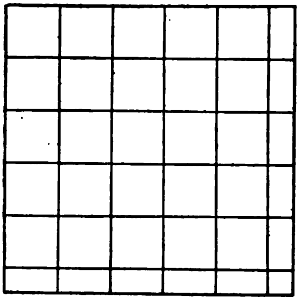
2. How long is the side of a square yard?

3. Draw a square foot on the blackboard.



Mark off the sides into inches and draw lines dividing it into inch squares as shown in the illustration. How many inch squares are there in one row an inch wide along one side? How many such rows are there in the square foot? A square foot, then, is equal to how many square inches?

4. Draw a square yard on the blackboard. Mark off each side of the square yard into square feet and draw lines, dividing the square yard into square feet. How many square feet are there in a row one foot wide along the side? How many such rows are there in a square yard? A square yard, then, is equal to how many square feet?



A rod is equal to $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet. A square rod is $5\frac{1}{2}$ yards long and $5\frac{1}{2}$ yards wide.

In a row 1 yard wide along one side there are $5\frac{1}{2}$ square yards. Since there are $5\frac{1}{2}$ such rows there are $5\frac{1}{2}\times5\frac{1}{2}$ square yards which are $30\frac{1}{4}$ square yards.

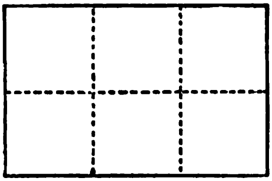
5. Measure on the floor of your school room or on the playground an area of 1 square rod. 160 square rods of area are called an *acre*.[1] An acre is the unit used in measuring land areas.

Table of Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd).
$30\frac{1}{4}$ square yards = 1 square rod
160 square rods = 1 acre

Exercise 4

1. Harold's garden plot is 49.5 feet long and 33 feet wide. How many feet are there in a rod? Find the length and width of his plot in rods.



2. A diagram may be drawn of his garden plot showing how it can be divided into square rods. How many square rods are there in one row a rod wide along one side? How many such rows are there in his plot? He may express the area of his garden plot as equal to how many square rods?

---

[1] In the olden time when the plowing was done with the slow and heavy oxen, the amount that a man could plow in a day was called an **acre.**

3. Harold raised peas and onions on his garden plot and made a profit of $9.24 on his plot of ground. He is measuring the plot to find how large it is so he can figure what his income would have been on an acre of ground. What was his average profit on each square rod of his garden?

4. Find how much he would have received from an acre of ground at that rate.

5. A square mile is how many rods long? How many rods wide? How many square rods are there in a row 1 rod wide along one side of a square mile? How many such rows are there in a square mile? A square mile, then, is equal to how many square rods?

6. How many square rods make one acre? How many acres are there in a square mile?

7. A square rod is 16½ feet long and 16½ feet wide. By means of a diagram, show how many square feet there are in a square rod.

8. How many square feet are there in an acre?

Suggestion: How many square feet are there in a square rod? (See Problem 7.) How manv square rods are there in an acre?

9. Lester[1] made a profit of $134.04 from his 109 square rods of sweet corn. What was his profit per square rod? How much would he have made on an acre at that rate?

10. Theodore[1] had a garden plot that had an area of 4356 square feet. He raised beans which brought $110.75 when sold in the market in the fall. His plot was what fraction of an acre? (See Problem 8.) What was his rate of profit per acre?

11. Ralph has a corn field 8 rods long and 3 rods wide. How many square rods are there in the field?

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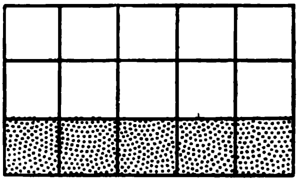
[1] From School Achievements, 1916.

Exercise 5. Areas of Rectangles



1. In a rectangle 5 inches long and 3 inches wide, how many square inches are there in the shaded row which is 1 inch wide?

2. How many such rows are there in the area of the whole rectangle? How many square inches are there in the area of the rectangle?

3. Draw[1] a rectangle 8 inches long and 5 inches wide. Find its area by the methods shown in Problems 1 and 2.

In the same way find the areas of rectangles:

4. 6 inches long and 4 inches wide.
5. 12 inches long and 9 inches wide.
6. 9 inches long and 7 inches wide.
7. 7 feet long and 5 feet wide.
8. 3 yards long and 2 yards wide.
9. 10 rods long and 8 rods wide.

From the preceding examples we see that:

The area of a rectangle is equal to the number of squares in a row (one linear unit in width) multiplied by the number of rows.

10. A kitchen is 12 feet long and 9 feet wide. Find the number of square feet in its area. That number of square feet equals how many square yards?

11. How much will it cost, at $1.15 per square yard, to cover the kitchen described in Problem 10 with linoleum?

---

[1] Cross section paper will save pupils a great deal of drawing in illustrating these problems. Each square can stand for 1 square inch, 1 square foot, etc. The outlines only of the rectangles will need to be drawn on the lines of the cross section paper.

12. Find the length of your schoolroom in feet. Find the width in feet. What is the area of the room in square feet? (Draw a diagram of the room, if necessary, to show how you obtain the area.)

13. A croquet ground is 54 feet long and 24 feet wide. How many square yards of ground does it contain?

14. A room is 15 feet long and 12 feet wide. How many square feet are there in the area of this room?

15. This room is partly covered by a rug 9 feet by 12 feet. Find the area of the rug in square feet. How much of the room is left uncovered by the rug?

16. The top of a certain type of individual school desk is 30 inches long and 24 inches wide. How many square feet are there in the area of this desk?

17. A billboard 30 feet long and 8½ feet high offers how much space for advertising purposes?

18. A square block measuring 338 feet each way is divided by two alleys crossing each other in the center of the block. The alleys are 8 feet wide. How many square feet do they contain? Make a diagram of the block, showing the alleys.

19. How many square feet does the block contain, including the alleys? How many, not counting the area of the alleys?

20. A rectangular field is 80 rods long and 40 rods wide. How many square rods does it contain? How many acres are there in this field?

21. How many acres are there in a rectangular field 40 rods long and 20 rods wide?

22. A farm is 160 rods long and 120 rods wide. How many acres does it contain?

Use local data to prepare supplementary problems.

Exercise 6. Measures of Weight

In buying, selling and shipping articles of merchandise, measures of weight are constantly used. In olden times the scales consisted of a horizontal bar, balanced on a pivot and having suspended from each end a shallow pan. In one of the pans was placed the object to be weighed and in the other was a standard measure of weight.[1]

Find how many different kinds of scales are used in weighing heavy articles by avoirdupois weight, such as groceries, farm products, meat, etc. There are four principal weights used in business, the ounce, the pound, the hundredweight, the ton.

Table of Avoirdupois Weight

16 ounces[2] (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton

The abbreviation for the word **ounce** was formerly o, the initial letter of the word. It was followed by an old mark which was used as a sign of abbreviation. This was mistaken for the letter Z which it resembled, and because of this mistake oz. became the abbreviation for **ounce.**

Cwt. is the abbreviation for hundredweight. What does the *C* mean? The *wt*?

---

[1] The standard for a pound was 7000 grains of wheat. As a matter of convenience people used a piece of lead which would exactly balance these grains. To weigh a half-pound, 3500 grains of wheat (or an equal weight of lead or some other substance) were used in the balance.

[2] Where articles are so valuable or so important that small fractions of an ounce must be considered, a system of weights other than Avoirdupois must be used. The druggist, in compounding medicines, frequently has to use a fraction of a grain. The druggist or apothecary uses a system of weights called Apothecaries' Weight. The jeweler in weighing gold and silver uses Troy Weights. See these tables in the back of the book.

The hundredweight is used in computing express and freight rates and in live stock quotations.

The abbreviation lb. for pound comes from a Latin word *libra,* meaning balance.

1. How many ounces are there in one-quarter of a pound?

2. One-half of a pound equals how many ounces?

3. How many ounces make $\frac{3}{4}$ of a pound?

4. Four pounds equal how many ounces?

5. Change 1 pound 2 ounces to ounces.

6. Change 40 ounces to pounds and ounces.

7. Since 7000 grains make a pound, Avoirdupois Weight, how many grains are there in an ounce?

Exercise 7

1. Mrs. Harvey bought 3 dressed chickens weighing 2 pounds 6 ounces; 2 pounds 9 ounces, and 2 pounds 5 ounces. How much did they cost her at 28 cents per pound?

2. If a ton of breakfast food is put up in 2-pound packages, how many packages will it fill?

3. Two tons of soap are packed in boxes holding 25 pounds each. How many boxes were filled from the two tons?

The *long ton* is used in custom houses and mines and consists of 2240 pounds.

4. If a coal dealer buys 100 long tons of coal at a mine and sells 100 regular tons of it to customers, how much coal does he have left?

5. A large grain boat built for the Great Lakes will carry 450,000 bushels of wheat. Wheat weighs 60 pounds per bushel. How many tons will this amount of grain weigh?

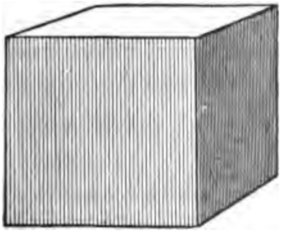
6. A certain make of piano, boxed, weighs 815 pounds. The freight rate between two cities on pianos is $.44 per hundredweight. Find the cost of shipping the piano by freight between those cities.

7. A farmer sold a car load of 19 cattle weighing, on the average, 1350 pounds each for $12.50 per hundredweight. How much did he receive for the cattle?

Exercise 8. Cubic Measure

These boys are studying the coal industry in their geography work. They were asked by the teacher to find how many tons of coal can be shipped in a flat car. They are measuring this coal car in order to solve the problem.

What kind of units of measure are used to measure the volume or capacity of a freight car, a box, a bin, a room, etc.? Perhaps you have already learned that we express volume by means of cubic units of measure.



A cube is a solid figure with 6 equal square faces. A cubic inch is an inch long, an inch wide and an inch high.

How long is a cubic foot? How wide. How high?

How long is a cubic yard? How wide? How high?

If I draw a square foot on the top of my desk, how many inch cubes does it take to cover this area so as to make a *layer* 1 inch high? How many such layers must I have to build a cubic foot? A cubic foot then equals how many cubic inches?

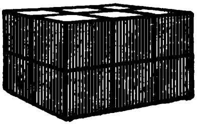
Draw a square yard on the floor. Divide it into square feet. How many cubic feet would it take to make a layer 1 foot high? How many cubic feet would you need to build up a cubic yard?

We are now ready to put these facts in the more convenient form of a table.

Table of Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord[1]

Exercise 9. Volume of a Rectangular Solid



A *rectangular solid* (or space) is a box-shaped solid with a rectangular bottom or base.

Examples of rectangular solids are boards, boxes, bins, freight cars, etc.

The first point to be noticed in finding the volume of a rectangular solid is that the number of cubic units in one layer on the base is the same as the number of square units in the base.

The next point is that the number of layers of cubes is the same as the number of linear units in the height.

1. Find the volume of a rectangular solid, 3 feet long, 2 feet wide and 2 feet high.

There are 6 square feet in the base. Then there are 6 cubic feet in one layer. In the 2 layers there are 2×6 cubic feet which are 12 cubic feet. The volume=12 cubic feet.

2. How many inch cubes does it take to build a rectangular solid 5 inches long, 4 inches wide and 4 inches high?

---

[1] The cord is used in measuring wood. It is a pile of wood 8 feet long, 4 feet wide and 4 feet high.

In the same way find the volumes of the following rectangular solids:

3. A box 6 inches long, 4 inches wide and 3 inches high.

4. A box 8 inches long, 5 inches wide and 4 inches high.

5. A box 4 feet long, 3 feet wide and 2 feet high.

From the preceding examples we see that:

The volume of a rectangular solid is the number of cubic units in one layer (a linear unit high) on the base multiplied by the number of layers in the height.

Exercise 10

1. The boys shown in the picture on page 135 found the coal car to be 37½ feet long, 8 feet wide and 4 feet deep. How many cubic feet were there in the volume of this car?

2. The boys also found that it took 38 cubic feet of soft coal to weigh a ton. How many tons of coal did they find the coal car would hold?

3. Measure the length, width and height of your school room. Find the number of cubic feet of space in the room.

4. It is estimated that a room should contain 500 cubic feet of space for each person who remains in it for a considerable length of time. For how many pupils will your room supply a suitable amount of air space?

5. If 1¼ cubic feet are allowed for a bushel of grain, how many bushels can be put in a bin 15 feet long, 10 feet wide and 6 feet deep?

6. A coal bin is 12 feet long, 8 feet wide and is filled to an average depth of 4 feet. How many tons of hard coal are there in it, allowing 35 cubic feet to one ton?

The cubic yard of earth, gravel or crushed stone is considered a *load* in figuring on excavations, building, hard roads, etc.

7. A cellar 12 feet long, 10 feet wide and 9 feet deep contains how many cubic feet? How many cubic yards? How many loads of dirt would be removed in making the excavation for this cellar?

Exercise 11. Wrapping Paper

Applied Problems in Weights and Measures

For convenience in tying up small packages much of the wrapping paper used is supplied in *rolls* attached to a rack at the end of a packing table, with a stiff wire holding down the outer fold which also serves to detach, easily and quickly, the paper in the desired lengths.

The standard rolls of wrapping paper are in 18-inch, 24-inch and 30-inch widths.

1. The weight of an 18-inch roll is 27 pounds. If the average linear foot of this paper weighs .6 of an ounce, what is the entire length of the paper in this roll in feet? In yards?

2. The weight of a 24-inch roll is 36 pounds. If the paper in this roll weighs .8 of an ounce per linear foot, what is the entire length of the paper in this roll in feet? In yards?

3. The weight of a 30-inch roll is 45 pounds. If the average weight of this paper is 1 ounce per linear foot, what is the entire length of the paper in this roll in feet? In yards?

4. At the opening of the school term Gertrude bought a reader, an arithmetic and some paper and pencils. The wrapping paper required to tie up this package measured 18 inches × 12 inches. How many such packages could be tied up from a full 18-inch roll?

5. At the beginning of the school term Harold bought a spelling book, a reader, an arithmetic, a geography and a writing tablet. These were tied up from a 24-inch roll, 16 inches being used. How many such packages could be wrapped from a full 24-inch roll?

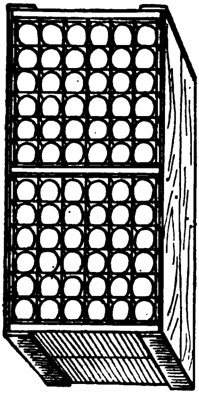
6. At the beginning of the school term Mr. Manning purchased an outfit of books and stationary supplies for his three children. This package was tied up with paper from a 30-inch roll, requiring 20 inches in length. How many such packages could be wrapped from a full 30-inch roll?

7. Make up a package of the books in your desk, find the length and width of the paper that would be necessary to wrap the same properly, and see which roll of paper could be used to the best advantage and how many such packages might be wrapped from a full roll of that width of paper.

For further reference to the paper industry see page 149 of the Advanced Arithmetic.

Exercise 12. Egg Fillers

Applied Review Problems combining fractions with long and square measure



1. Have you noticed at your grocery store the little strawboard compartments, or "fillers," which enable eggs to be shipped to the city markets without breaking? These fillers, as shown in the illustration, hold 3 dozen eggs each. The wooden case in which the shipments are made is long enough to hold 2 fillers and deep enough for 5 layers of fillers. This makes how many fillers for each shipping case, and room for how many dozen eggs in each full case?

2. These trays, or "fillers," are 11½ inches square. How many square inches does that make?

3. From the picture it will be seen that each filler is made by uniting seven strips one way with seven strips the other way, each 11½ inches. The combined length of these strips is how many feet?

4. If these strips are 2¼ inches wide, how many square inches of strawboard will it take to make a filler?

5. At the top and bottom and between each layer of fillers is placed a single piece of strawboard of the proper size to fit the inside of the shipping case. How large are these pieces and how many are needed to make a bottom and a cover for each filler in a completed egg case?

6. A freight car will carry 10 tons of baled straw. This is equivalent to 6 tons of finished strawboard. What part of a car load of straw is required to produce 1 ton of strawboard?

7. If straw in bales, delivered to the factory, costs $9.00 per ton, what will be the cost of the straw for 1 ton of strawboard?

8. From 1 ton of strawboard may be manufactured 670 cases of egg fillers. How many fillers will a car load of straw produce?

9. The annual output of egg trays, or fillers, of a certain factory is estimated to be sufficient to hold 150,000,000 dozen eggs. If each filler holds 3 dozen eggs, how many fillers would this factory produce during the year?

10. How many tons of strawboard would it require to produce the fillers for this number of cases?

11. It requires 2¾ feet of cotton wood lumber to make the shipping cases for the egg fillers. How many feet of lumber would be needed to make the number of shipping cases indicated in Problem 9?

PART II

CHAPTER I

Training in Efficiency

Courtesy International Harvester Co.

The above illustration shows a rural canning club at work under the direction of experts. The members of this club are training themselves to be efficient in the home by learning to reduce the high cost of foodstuffs.

Efficiency in any line of work requires both *knowledge* of how to do the work and *skill* in the necessary operations. These pupils are not only acquiring skill in the process of canning fruits and vegetables, but they are also computing the cost of each can and, by comparison with the retail prices in stores, they are able to estimate the amount saved in this way.

To solve all the problems that arise in such work, these pupils must know how to use weights and measures, how to compute the amount and rate of profit and be able to do the fundamental processes of integers and fractions rapidly and accurately.

Speed and accuracy in the use of these fundamental processes can be attained by every pupil with sufficient practice. Many practice exercises are included in the beginning of this year's work in order to help you acquire the necessary rapidity and accuracy in your computations.

Exercise 1. Reading Numbers

Most persons have little need to read numbers larger than thousands in the arithmetic work relating to their own business and to matters of the home. For some purposes, however, large numbers must be used.

Bring to the class selections from books, magazines and papers containing large numbers. Read these numbers to the class and explain how they were used.

You have doubtless heard of the Panama-Pacific International Exposition, held at San Francisco in 1915. Perhaps some of you or your parents were there.

Read the attendance at each of the great expositions held in the United States since 1876.

Designation	Place	Year	Attendance
International Centennial	Philadelphia	1876	9,910,966
World's Columbian	Chicago	1893	27,539,014
California Mid-Winter	San Francisco	1894	2,250,000
Cotton States	Atlanta	1895	1,179,889
Tennessee Centennial	Nashville	1897	1,886,706
Trans-Mississippi	Omaha	1898	2,613,508
The Pan-American	Buffalo	1901	8,120,048
The Louisiana Purchase	St. Louis	1904	19,694,855
The Lewis and Clark	Portland	1905	2,545,509
The Alaska-Yukon-Pacific	Seattle	1909	3,740,499
The Panama-Pacific International Exposition	San Francisco	1915	13,127,103
The Panama-California International Exposition	San Diego	1915	2,050,030

You will notice that the numbers in the above table are separated into groups of three figures for convenience in reading. These groups are generally called periods. To separate a whole number into periods, begin with the right-hand figure and point off the first three figures by a comma and continue in the same way until three or less figures are left in the left-hand period.

Exercise 2

All of the numbers on the preceding page are limited to three periods. Some numbers require more than three periods. For example: read the following number, which gives the number of bushels of corn produced in the United States in 1916:

$$\overset{\text{Billions}}{2},\overbrace{710}^{\text{Millions}},\overbrace{000}^{\text{Thousands}},\overbrace{000}^{\text{Units}}$$

This number is read two billion, seven hundred ten million.

There are no figures in the units and thousands periods except zeros. It is necessary to fill in these periods with zeros, however, in order that the 2 may stand in the fourth period and thus represent billions.

Read the following statements:

1. The national wealth of the United States is estimated by the census bureau in 1916 at $187,739,000,000, or $1,965 for each man, woman and child in the country.

2. The total production of cotton in the United States in 1916 was 5,580,500,000 pounds.

3. The crude petroleum produced in the United States in 1916 amounted to 16,806,372,368 gallons.

4. The total amount on deposit in the savings banks in the world is $18,081,239,620.

Exercise 3. Drill in Reading Numbers

Read the following numbers:

1. 2,103. **2.** 1,001. **3.** 30,226. **4.** 279,000. **5.** 280,010.
6. 1,763,000. **7.** 2,807,296. **8.** 2,000,017. **9.** 22,000,962.
10. 845,623,000. **11.** 33,870,000. **12.** 210,000,606.
13. 340,721,000. **14.** 1,900,823,000. **15.** 21,980,000,000.

EXERCISES FOR SPEED AND ACCURACY

Addition, Subtraction, Multiplication and Division

To the Teacher: These exercises are continued from page 38 and should follow a review of those exercises. The time limit for the sixth grade should be ½ minute less than that allowed in the fifth grade. If this time limit for the sixth grade (2¼ minutes) proves too low or too high for your particular class, adjust it to meet the needs of your class. Use the same time limit on each exercise.

Instead of concentrating all of the time on these exercises until they are finished, a short drill period should be given at the beginning of each recitation. Experiments in drill work have shown that short drill periods, distributed over a long period of time, produce greater gains than long drill periods concentrated in a short period of time.

See the suggestions on page 33 for two methods of using these practice exercises.

Exercise 4. Multiplication

48	57	72	85	92	83	50	76
30	40	50	20	60	70	80	40

84	35	48	53	71	38	93
90	60	40	80	90	20	30

Exercise 5. Addition

38	67	32	21	36	30	74	72
64	57	54	43	29	63	44	78
34	43	76	97	60	75	61	51
78	86	89	66	83	27	20	65
95	95	54	10	40	81	57	36

These exercises are intended for short, rapid drills at the beginning of each recitation period.

Exercise 6. Division

2)430	5)895	3)171	2)642	5)325
6)474	9)756	4)372	2)198	8)472
3)522	9)468	7)259	3)147	6)888

Exercise 7. Addition

111	635	326	784	869
222	340	362	215	120
362	441	150	655	252
425	236	228	132	525
631	725	795	438	651
225	262	203	351	325
521	573	435	726	522
420	315	323	252	175

Exercise 8. Subtraction

745	473	763	394	223
48	51	86	62	58
624	250	415	432	848
55	30	97	21	59
421	555	317	590	500
38	35	58	60	73

Exercise 9. Addition

127	423	377	265	188	177	509
346	369	208	127	506	718	306
223	351	746	462	119	227	148
547	329	136	228	239	136	226

Exercise 10. Multiplication

427	342	872	951
12	27	36	27

Exercise 11. Subtraction

678	465	546	388	394	189
354	123	124	235	181	145

857	311	385	618	621	785
640	200	352	303	611	463

157	141	383	468	124
133	120	202	357	114

Exercise 12. Addition

34	73	29	19	22	25
23	29	28	77	43	72
26	93	35	81	46	84
11	58	15	52	24	20
53	54	93	51	70	12
17	13	20	30	83	64
56	16	47	48	21	27

Exercise 13. Division

20)600	30)900	40)120	50)550
70)490	60)300	80)240	20)800
40)280	70)630	60)420	80)400
30)270	80)480	80)640	50)250
20)140	90)810	70)350	20)120
50)700	70)280	50)300	60)660
80)800	90)630		50)750

Exercise 14. Subtraction

579	856	634	755	384
445	587	312	557	272
227	883	359	472	786
105	595	244	186	555
449	200	326	791	327
228	185	222	693	207

Exercise 15. Addition

34	42	85	69	86
17	88	24	83	35
91	89	55	14	43
63	28	67	22	33
49	29	95	47	56
30	44	70	98	67
62	20	85	73	35
74	88	60	25	20
83	92	37	40	45

Exercise 16. Addition

764	843	597	634	488
886	568	488	397	536
927	848	368	456	543
384	153	645	844	567

Exercise 17. Multiplication

452	572	147
121	251	362

Exercise 18. Addition

288	167	472	144	263	321
326	221	149	186	175	490
281	482	201	214	199	129

161	255	221
185	245	178
127	105	102

Exercise 19. Division

89)8722 84)7644 37)2442

21)2625 84)6804

Exercise 20. Addition

645	524	249	626
256	190	462	257
192	309	318	199

488	676	507	496
262	166	345	605
413	234	159	619

Exercise 21. Subtraction

753	228	374	783	831
386	169	185	299	564

823	912	973	211	243
337	864	699	186	178

433	741	256	555
277	666	167	478

Exercise 22. Addition

971	456	998	878	246	751
783	326	875	717	202	344
111	411	502	616	115	561
888	600	751	135	494	228
438	812	209	802	787	794

Exercise 23. Multiplication

2344	688	3271
689	75	422

Exercise 24. Addition

422	492	555	862
567	835	122	545
895	976	263	535
361	524	962	262
227	327	165	213
426	831	447	991
772	991	887	104

Exercise 25. Division

$33\overline{)792}$ $\quad$ $72\overline{)16992}$ $\quad$ $21\overline{)378}$

Exercise 26. Addition

457	717	617	
913	308	297	
124	890	434	
718	383	456	
323	262	913	
444	999	363	422
353	424	802	560
232	282	791	789
666	415	345	671

Exercise 27. Subtraction

865341	723597	583121	895752
46233	83701	76243	59863

400271	687094	284463
38105	54321	18989

Exercise 28. Division

31)868 999)22977 39)936

Exercise 29. Subtraction

357912352	975432186	545123615
87527286	78523321	80828268

934105526	773642
65687810	55566

SUPPLEMENTARY EXERCISES

These exercises are provided for pupils who finish the regular practice exercises. They are purposely made more difficult in order to offer a special incentive to rapid workers. The same time limits should be used for these exercises.

Exercise 30. Addition

2	5	2	1	7	8	8	7
3	3	2	3	5	8	9	5
4	4	5	8	1	2	5	5
9	2	4	6	7	3	5	9
3	1	2	8	6	2	1	2
4	4	1	7	9	3	3	3
8	9	6	1	3	6	7	8
7	5	0	4	1	7	9	9
6	3	7	7	5	0	4	6
5	7	9	4	1	7	2	3
2	8	6	5	9	8	4	3

Exercise 31. Subtraction

8345	4206	9378	6500
6237	2973	6753	1828
2175	4256	8835	7000
1642	1961	4995	5284
5803	6082	3734	8312
2768	4766	1887	3123

Exercise 32. Multiplication

6328	1976	5836	8219
2	9	7	3
6785	3628	9163	7658
8	5	6	4
3784	2178	6539	6589
9	3	8	7

Exercise 33. Addition

567	913	422	308
457	717	637	415
492	835	124	383
232	286	772	991
895	976	718	890
353	408	426	831
561	524	323	262
927	873	496	999
807	573	758	926

Exercise 34. Multiplication

6293	3829	4573	8165
27	64	38	95

Exercise 35. Division

2)7652	9)82503	7)26992	3)8673
8)46288	5)31415	6)36834	4)24728
3)3834	2)4906	9)45324	7)4893

Exercise 36. Subtraction

937452	483592	257634	532000
362183	217926	108926	371483
871039	558372	783219	593042
714826	219387	438478	217562

Exercise 37. Division

$72\overline{)30816}$	$44\overline{)1584}$	$58\overline{)36366}$

Exercise 38. Addition

8073	3468	7504	2935	3884
5846	5921	2836	6845	1549
1279	8705	9548	1906	6331
6331	2499	6342	2783	5276
7628	6574	7807	5619	9745

Exercise 39. Review Problems

Add:

1.	2.	3.	4.
$88.45	$142.70	$100.00	$290.50
45.56	75.85	63.75	142.35
35.04	67.08	91.43	624.85

5. A farmer borrowed $1250. He paid back $150 at one time and $375 at another. How much did he have left to pay?

6. If you go to the store with $2 and buy 5 pounds of sugar at 8½ cents; a can of peas for 12 cents; 2 cakes of yeast at 1 cent each and 1 pound of coffee for 28 cents, how much change should you receive?

7. Mrs. Mathews had the following meat bill for 1 week: 5 pounds of beef steak at 27 cents; 3 pounds of lard at 20 cents; 1½ pounds bacon at 34 cents; 4 pounds soup meat at 14 cents; 3½ pounds pork-loin at 25 cents. What change should she receive from a $5 bill when she settles this account?

8. Ellen bought the following material to make a dress: 6 yards flowered dimity at 25 cents; 1 spool thread at 5 cents; 10 yards lace edging at 8 cents; 6 yards insertion at 9 cents; 1 dozen buttons for 10 cents; 2½ yards ribbon at 30 cents. What change did she receive from 4 one-dollar bills?

9. George bought the following groceries for his mother: 3 grapefruits at 10 cents; 2 cans of tomatoes at 15 cents; 5 pounds of sugar for 35 cents; 1 package oatmeal for 10 cents; 1 can salmon for 25 cents and 3 pounds of rice at 8 cents. He gave the cashier three one-dollar bills. How much change did he receive?

10. A farmer sold 33 hogs averaging 228 pounds each in weight at $15.25 per hundred pounds. How much did he receive for the hogs?

Exercise 40. Poultry

1. The number of eggs produced by a hen in a common flock is estimated at 70 eggs a year. How much are these eggs worth at 25 cents a dozen?

2. Where the best methods of selecting and feeding poultry have been followed, one hen has produced 303 eggs in a year. How much were the eggs of this hen worth at 25 cents a dozen?

3. If 30 Brahma hens produce an average of 10 dozen eggs each per year, what will be the weight of the eggs produced by them in a year, each egg weighing $\frac{1}{8}$ of a pound?

4. If 30 Leghorn hens produce an average of 150 eggs each per year, the eggs averaging 10 to the pound, what will be the total weight of the eggs produced in a year by this flock?

5. Four Pekin ducks produced 8 dozen eggs, each, in one season, and 4 other ducks produced 380 eggs. What was the total number of eggs produced by this flock of 8 ducks?

6. The standard weight of Plymouth Rock hens is $7\frac{1}{2}$ pounds and of roosters $9\frac{1}{2}$ pounds. Find the total weight of 10 hens and 40 roosters averaging the standard weight. How much would these chickens bring at 14 cents a pound?

7. The standard (or average) weight of Bronze turkeys is 26 pounds. How much will a farmer receive for 20 Bronze turkeys of standard weight at 22 cents a pound?

8. The standard weight of Wyandotte hens is $6\frac{1}{2}$ pounds and of roosters $8\frac{1}{2}$ pounds. How much less will 10 Wyandotte hens and 40 Wyandotte roosters weigh than the same number of Plymouth Rock hens and roosters? (See Problem 6.)

9. If 10 turkey hens produce an average of 24 eggs each; if $\frac{3}{4}$ of the eggs hatch and $\frac{4}{5}$ of the young turkeys live and grow to an average weight of 20 pounds each, what will be the total weight of the young turkeys raised from the flock of 10 hens?

Exercise 41. Bees

1. A bee keeper with 40 hives secures from each an average of 48 pounds of comb honey in a year. If he sells this honey to a grocer at $12\frac{1}{2}$ cents a pound, what are his yearly receipts from the 40 hives?

2. How much more would he have received if he had retailed it himself at 20 cents a pound, delivered?

3. In a winter a colony of bees uses 30 pounds of honey. If the bee keeper prepares a substitute made of granulated sugar at 7 cents a pound, what will be the saving for each hive if the honey is sold at 15 cents a pound?

4. Cases of unseparated honey average not less than 23 pounds per section. How much would a bee keeper receive for 12 sections at 15 cents per pound?

5. An apiary of 94 colonies of bees produced 10,600 pounds of honey in a year. The owner sold the honey for $1060. What did he receive per pound for the honey? What was the average production for each colony?

6. A colony of bees consisted of a queen bee, 300 drones and about 10 times as many workers. After the drones were all killed and $\frac{1}{4}$ of the workers lost, how many bees were left in the colony for the winter season?

7. Granulated honey is made into one-pound bricks and wrapped in paraffin paper. Find the amount received for 110 bricks of granulated honey at 25 cents each.

8. A standard honeycomb frame for bee hives is $17\frac{5}{8}$ inches long and $9\frac{1}{8}$ inches broad. How many square inches are there in the area of one of these frames? (See page 129.)

Reserve Practice Problems in Column Addition

Exercise 42

1.	2.	3.	4.	5.	6.	7.
729	636	252	547	869	329	755
171	993	489	312	945	469	934
595	484	825	673	739	279	216

Exercise 43

1.	2.	3.	4.	5.	6.	7.
628	476	425	354	635	661	398
442	262	538	324	673	827	278
727	754	677	946	137	573	667

Exercise 44

1.	2.	3.	4.	5.	6.	7.
859	869	968	945	897	579	548
836	339	429	714	782	281	697
743	267	658	565	153	438	671

Exercise 45

1.	2.	3.	4.	5.	6.	7.
958	693	948	985	846	595	962
438	282	347	954	679	563	834
434	456	430	632	774	105	220
253	532	677	846	329	961	704

Exercise 46

1.	2.	3.	4.	5.
23489	96324	47368	17263	59584
65342	85963	93475	72197	76340
18964	68741	82943	15650	48279
43718	49783	21062	52586	95543

Exercise 47. Supplying the Balance

1. What number added to 54+36+48+19 will equal 300?

54	300
36	157
48	143
19	
157	

This problem may be solved in two ways. The four numbers given may be added together and their sum subtracted from 300.

Another way is to write the four numbers above the total, leaving a blank space for the missing number.

The numbers may now be added downward from the top and the necessary figure may be supplied to equal the figure in the total. Thus, in adding units' column from the top, we say four ten, eighteen, twenty-seven and then supply three to make up the ten indicated in the total. In adding ten's column, we say three, eight, eleven, fifteen, sixteen and then supply four to make up the twenty indicated in the total. In hundreds' column, one must be added to the two "carried" and supplied to equal the three in the total. The balance, then, is 143.

54
36
48
19
300

Which of these methods do you prefer to follow in problems like the one given above? Why? What is the saving of figures in the second form of the work?

Supply the balance in the following problems:

2.	3.	4.	5.	6.	7.
23	19	27	83	43	15
59	38	58	28	75	34
17	25	32	90	19	97
24	14	44	45	34	26
258	99	564	298	215	239

8.	9.	10.	11.
1806	29,461	93,170	21,034
2970	37,286	208,109	79,281
3106	54,972	392,860	46,178
10,415	249,862	864,213	240,179

12. Of a city library of 2600 books, 350 were in circulation, 217 were in the repair shop and 114 were withdrawn from circulation. What number remained subject to call? Tabulate the items and supply the balance.

In circulation	350
In repair shop	217
Withdrawn	114
Balance	
Total	2600

To tabulate the items is to state them in order under appropriate headings, with the accompanying figures in columns as shown at the left. Find the balance.

Tabulate the items and supply the balance in the following problems.

13. Of 826 chickens a poultryman sold 216 in June, 224 in July and 219 in August; 10 were stolen, 6 presented to friends and 20 used at home. How many did he have left?

14. A girl having 3 dozen oranges gave 18 to her classmates, 5 to her brother and 3 to a neighbor girl. How many did she have left?

15. After a battle, out of a company of 100 soldiers, 68 were present at roll call, 2 were absent on duty, 11 were reported killed, 16 were reported wounded and the remainder were unaccounted for. How many missing men were unaccounted for?

16. Of a charity fund of $98.50, $32.50 was expended for coal, $18 for bread, $6.90 for milk, $4.20 for meat and $9.40 for medicines. Find the balance left on hand.

17. Richard received an allowance of $10 for this month. He has spent $2.15 for carfare, $.65 for stationery and postage; $2.05 for amusements and 50 cents for charity. What balance has he left?

18. In a certain city there were 780 children of school age who were not enrolled in the public school. Of this number 235 were reported employed, 417 enrolled in private schools, 17 ill and 14 absent from home. What was the balance unaccounted for?

Exercise 48

What is the test of correctness of work in subtraction? To what must the sum of the subtrahend and the difference always be equal? Perform these subtractions.

1.	2.	3.	4.
2,987,634	11,000,000	70,807,632	12,103,205
1,098,745	8,939,393	49,988,753	9,008,096

5.	6.	7.	8.
3,000,003	9,827,634	22,987,598	31,313,138
1,888,995	8,848,746	19,998,879	13,131,399

Exercise 49

Subtract:

1.	2.	3.	4.	5.	6.	7.
999	432	845	897	721	803	348
156	147	321	624	632	256	159

8.	9.	10.	11.	12.	13.	14.
567	725	886	1720	492	1556	1336
321	647	413	765	386	948	799

Exercise 50. Cattle Raising

Country	Year	Total Cattle
British India....	1909	108,000,000
United States...	1910	71,000,000
Russia.........	1908	47,000,000
Argentina......	1908	29,000,000
Brazil..........	1908	25,000,000
Germany.......	1907	21,000,000
Austria-Hungary	1908	18,000,000
France.........	1909	14,000,000
United Kingdom	1910	12,000,000
Australia.......	1909	11,000,000
Canada........	1910	7,000,000
Other Countries.	....	85,000,000

1. Find the total number of cattle in the world between 1908 and 1910 as shown in the table.

2. How many more cattle were there in British India than in the United States?

3. How many more cattle were there in the United States than in Russia?

4. How many more cattle were found in Germany in 1907 than in France in 1909?

Exercise 51. Multiplication

1.	**2.**	**3.**	**4.**	**5.**
9634	5278	6075	9128	5367
8	7	4	6	3
6.	**7.**	**8.**	**9.**	**10.**
9834	7562	4180	5562	8394
2	5	9	8	6
11.	**12.**	**13.**	**14.**	**15.**
2751	8396	4705	6219	3574
7	2	5	9	4
16.	**17.**	**18.**	**19.**	**20.**
8845	9065	1438	7650	4983
6	3	9	7	8

Exercise 52. Multiplication

1.	**2.**	**3.**	**4.**	**5.**
6382	4759	1806	3724	9560
91	73	48	56	38
6.	**7.**	**8.**	**9.**	**10.**
5568	3945	2713	8465	9207
92	63	87	44	15
11.	**12.**	**13.**	**14.**	**15.**
6185	2347	9408	5277	9314
26	89	37	54	68
16.	**17.**	**18.**	**19.**	**20.**
8623	7144	9578	6624	8357
139	508	291	746	805

Exercise 53. Review Problems in Short Division

1. $5\overline{)8435}$
2. $9\overline{)6255}$
3. $3\overline{)7423}$
4. $8\overline{)6328}$
5. $2\overline{)9503}$
6. $7\overline{)7364}$
7. $4\overline{)7216}$
8. $6\overline{)8137}$
9. $5\overline{)3045}$
10. $9\overline{)2853}$
11. $3\overline{)5616}$
12. $8\overline{)9243}$
13. $2\overline{)4650}$
14. $7\overline{)1043}$
15. $4\overline{)5692}$
16. $6\overline{)9024}$
17. $5\overline{)3153}$
18. $9\overline{)8379}$
19. $3\overline{)4566}$
20. $8\overline{)2182}$
21. $2\overline{)5634}$
22. $7\overline{)4228}$
23. $4\overline{)9316}$
24. $6\overline{)2358}$
25. $5\overline{)8730}$

Exercise 54. Review Problems in Long Division

1. $67\overline{)23115}$
2. $54\overline{)43578}$
3. $93\overline{)87215}$
4. $28\overline{)15036}$
5. $55\overline{)51040}$
6. $37\overline{)10878}$
7. $68\overline{)39426}$
8. $81\overline{)51354}$
9. $94\overline{)23594}$
10. $52\overline{)73156}$
11. $36\overline{)13212}$
12. $87\overline{)79692}$
13. $34\overline{)28084}$
14. $59\overline{)83156}$
15. $78\overline{)26676}$
16. $63\overline{)11655}$
17. $74\overline{)44918}$
18. $28\overline{)16284}$
19. $96\overline{)70392}$
20. $53\overline{)43195}$
21. $81\overline{)51192}$
22. $56\overline{)10756}$
23. $39\overline{)27456}$
24. $77\overline{)41810}$
25. $48\overline{)29136}$
26. $91\overline{)28847}$
27. $26\overline{)18305}$
28. $73\overline{)21462}$

Exercise 55. Live Stock

Courtesy International Harvester Co.

1. In 1916 there were 21,166,000 horses in the United States valued at $2,150,468,000. Find the average value of each horse.

2. In the same year there were 4,565,000 mules with an average value of $113.87. Find the total value of the mules for that year.

3. By what amount did the average value of a mule exceed the value of a horse?

4. The number of milch cows was 21,988,000, with a total valuation of $1,185,119,000. Find the average value of a milch cow for that year.

5. There were 39,453,000 other cattle with an average value of $33.49. Find the total value of these cattle.

6. Find the total number and the total valuation of all the cattle in the United States for that year, including milch cows.

7. The total number of sheep was 49,162,000 valued in 1916 at $254,348,000. What was the estimated value of each sheep in that year?

8. The valuation of the 68,047,000 hogs in that year was $571,890,000. Find the average value of a hog based upon those estimates.

9. How much more was the value of a hog than the value of a sheep?

10. A milch cow was considered equal in value to how many hogs?

Exercise 56. Vegetables and Grains

1. In 1915 there were 3,761,000 acres of land planted in potatoes. The total yield was 359,103,000 bushels. Find the average yield per acre.

2. The total valuation of these potatoes at digging time was $221,104,000. What was the value per bushel?

3. In 1916 the average yield per acre was 94.2 bushels. The number of acres planted was approximately 3,377,000. What was the total yield of potatoes for 1916?

4. How many less bushels of potatoes were raised in 1916 than in 1915?

5. By how many bushels per acre did the average yield for 1916 exceed that for 1915?

6. How many more acres were planted in 1915 than in 1916?

7. It requires about 10 bushels of seed potatoes to plant an acre. How many bushels of seed potatoes were required to plant the 1916 crop?

8. Stewart sold 11 bags of onions at $1.50 per bag, 5 bags at $1.75 per bag and 100 bags at $2 per bag. How much were his receipts from his onions?

9. Clement raised 17 bushels of potatoes. He sold 5 bushels at $1.20 a bushel and the remainder at $1.70 a bushel. How much did he receive for his crop of potatoes?

10. Herman sold the vegetables which he raised as follows: 13½ bushels of tomatoes at $1 per bushel; 19 quarts of beans for $2.17 and 3½ bushels of potatoes for $6.30. How much profit did he make from his garden if his expenses were $1.85?

11. In 1915 the total yield of corn in the United States was 3,054,535,000 bushels. The number of acres planted in corn that year was 108,321,000. Find the average yield per acre.

12. The value of the 1915 corn crop at husking time was estimated at $1,755,859,000. What was the value of the corn per bushel? What was the average value of an acre of corn in that year?

13. The average yield of corn per acre in 1916 was only 24.9 bushels. The total yield amounted to 2,710,000,000 bushels. How many acres were planted in corn in 1916?

14. On 59,898,000 acres 1,011,505,000 bushels of wheat were raised in 1915. What was the average yield per acre?

15. In 1916 the total yield of wheat was 611,000,000 bushels with an average yield of only 12 bushels per acre. How many acres of wheat were raised in 1916?

16. By how many bushels did the 1915 crop of wheat exceed the crop of 1916?

17. How many bushels greater was the yield of wheat per acre in 1915 than in 1916?

18. How many more acres of wheat were planted in 1915 than in 1916?

19. In 1915 the yield of oats on 40,780,000 acres was 1,540,000,000 bushels. What was the average yield per acre?

20. In 1916 the oats yield was 1,231,000,000 bushels. How many bushels less was this yield than that for 1915?

21. Find the total number of bushels of corn, wheat and oats for 1915. Find the total yield for these three grains in 1916. By how much does the total yield for the three grains in 1915 exceed the total yield in 1916? Other conditions being equal, how would the prices for 1916 compare with those for 1915?

Exercise 57. Sheep and Wool

1. In a recent year there were fifteen states in which there were over a million sheep. Find the total number in these fifteen states:

Wyoming	4,338,000	Oregon	2,563,000	Colorado	1,839,000
Montana	3,941,000	California	2,450,000	Nevada	1,532,000
New Mexico	3,440,000	Texas	2,156,000	Missouri	1,416,000
Idaho	3,102,000	Utah	2,089,000	Iowa	1,274,000
Ohio	3,067,000	Arizona	1,849,000	Indiana	1,058,000

2. The total number of sheep in the United States in that year amounted to 49,162,000. How many sheep were there in the other 30 states in our country?

3. The total production of raw wool for the same year amounted to 288,777,000 pounds. This amount was taken from 36,698,000 sheep. What was the average number of pounds of wool obtained from a sheep?

4. The production of raw wool in Wyoming was 29,040,000 pounds and in Montana, the nearest competitor, 28,682,000 pounds. By how much did the production in Wyoming exceed that in Montana?

Exercise 58. Coal and Iron

1. During a recent year the total production of iron in the United States was 29,916,213 long tons of 2240 pounds. Find the total production of iron in pounds.

2. The production of iron in four leading states for that year was as follows: Pennsylvania 12,790,668 tons; Ohio 6,912,962 tons; Illinois 2,447,220 tons; Alabama 2,049,453 tons. Find the total production, in tons, for these four states.

3. How many tons of iron were produced during that year in the remaining 41 states?

4. How many more tons of iron were produced in Pennsylvania that year than in Ohio, Illinois and Alabama combined?

5. In 1915 there were produced in the United States approximately 4,357,012,800 pounds of steel rail. How many tons of 2240 pounds?

6. In a certain year 377,414,259 tons of bituminous (soft) coal were mined. The value of this coal was estimated at $493,309,244. What was the average value per ton?

7. During the same year there were 81,090,631 tons of anthracite (hard) coal mined, having a value of $188,181,399. What was the average value of the anthracite coal?

8. Find the total number of tons of both bituminous and anthracite coal produced that year.

9. Find the total value of both anthracite and bituminous coal for that year.

Review Exercises in Fractions

To the Teacher: The following exercises have been prepared from speed tests taken by a large number of pupils. They are graded so that it should take the average sixth-grade class 2 minutes to solve each exercise. Pupils should practice upon them until they can do each exercise in that time or less.

Any process which has been forgotten may be reviewed by reference to Chapter III in Part I.

These exercises should be hectographed to save the pupils' time in copying.

Exercise 59

Supply the missing numerators:

1. $\frac{1}{2}=\frac{}{14}=\frac{}{8}=\frac{}{6}=\frac{}{10}=\frac{}{4}=\frac{}{12}=\frac{}{16}$.
2. $\frac{1}{3}=\frac{}{36}=\frac{}{12}=\frac{}{9}=\frac{}{18}=\frac{}{15}=\frac{}{6}=\frac{}{24}$.
3. $\frac{1}{4}=\frac{}{12}=\frac{}{8}=\frac{}{16}=\frac{}{36}=\frac{}{24}=\frac{}{28}=\frac{}{32}$.
4. $\frac{1}{5}=\frac{}{15}=\frac{}{30}=\frac{}{10}=\frac{}{25}=\frac{}{35}=\frac{}{20}=\frac{}{40}$.
5. $\frac{1}{6}=\frac{}{24}=\frac{}{12}=\frac{}{42}=\frac{}{18}=\frac{}{48}=\frac{}{30}=\frac{}{36}$.
6. $\frac{1}{7}=\frac{}{49}=\frac{}{14}=\frac{}{28}=\frac{}{35}=\frac{}{21}=\frac{}{42}=\frac{}{56}$.
7. $\frac{1}{8}=\frac{}{64}=\frac{}{24}=\frac{}{48}=\frac{}{16}=\frac{}{32}=\frac{}{56}=\frac{}{40}$.
8. $\frac{1}{9}=\frac{}{18}=\frac{}{36}=\frac{}{27}=\frac{}{54}=\frac{}{63}=\frac{}{45}=\frac{}{81}$.

Exercise 60

Supply the missing numerators:

1. $\frac{2}{3}=\frac{}{9}=\frac{}{18}=\frac{}{24}=\frac{}{6}=\frac{}{15}=\frac{}{12}$.
2. $\frac{3}{4}=\frac{}{32}=\frac{}{8}=\frac{}{16}=\frac{}{24}=\frac{}{12}=\frac{}{20}$.
3. $\frac{3}{5}=\frac{}{15}=\frac{}{10}=\frac{}{25}=\frac{}{30}=\frac{}{20}=\frac{}{40}$.
4. $\frac{5}{6}=\frac{}{42}=\frac{}{12}=\frac{}{18}=\frac{}{24}=\frac{}{30}=\frac{}{48}$.
5. $\frac{3}{8}=\frac{}{24}=\frac{}{48}=\frac{}{16}=\frac{}{64}=\frac{}{32}=\frac{}{40}$.

Exercise 61

Change to lowest terms:

1. $\frac{4}{12}=$
2. $\frac{15}{24}=$
3. $\frac{5}{15}=$
4. $\frac{9}{18}=$
5. $\frac{4}{6}=$
6. $\frac{6}{8}=$
7. $\frac{16}{20}=$
8. $\frac{24}{30}=$
9. $\frac{5}{10}=$
10. $\frac{4}{8}=$
11. $\frac{12}{36}=$
12. $\frac{10}{15}=$
13. $\frac{16}{24}=$
14. $\frac{3}{6}=$
15. $\frac{2}{8}=$
16. $\frac{16}{32}=$
17. $\frac{12}{16}=$
18. $\frac{12}{15}=$
19. $\frac{6}{15}=$
20. $\frac{10}{20}=$
21. $\frac{2}{4}=$
22. $\frac{2}{6}=$
23. $\frac{6}{9}=$
24. $\frac{32}{36}=$

Exercise 62

Change to improper fractions:

1. $2\frac{3}{5}=$
2. $3\frac{4}{7}=$
3. $6\frac{1}{4}=$
4. $8\frac{3}{8}=$
5. $7\frac{1}{2}=$
6. $1\frac{7}{8}=$
7. $4\frac{5}{8}=$
8. $3\frac{5}{9}=$
9. $11\frac{2}{3}=$
10. $6\frac{1}{5}=$
11. $8\frac{3}{4}=$
12. $7\frac{2}{5}=$
13. $2\frac{9}{11}=$
14. $5\frac{4}{5}=$
15. $5\frac{3}{8}=$
16. $7\frac{2}{3}=$
17. $6\frac{3}{7}=$
18. $9\frac{1}{2}=$
19. $3\frac{8}{9}=$
20. $7\frac{4}{9}=$
21. $9\frac{5}{6}=$
22. $5\frac{1}{3}=$
23. $3\frac{7}{12}=$
24. $4\frac{1}{3}=$
25. $3\frac{2}{5}=$

Exercise 63

Change to integers or mixed numbers:

1. $\frac{11}{3}=$
2. $\frac{17}{9}=$
3. $\frac{20}{5}=$
4. $\frac{45}{8}=$
5. $\frac{17}{5}=$
6. $\frac{17}{6}=$
7. $\frac{42}{3}=$
8. $\frac{43}{7}=$
9. $\frac{19}{4}=$
10. $\frac{30}{7}=$
11. $\frac{38}{11}=$
12. $\frac{18}{9}=$
13. $\frac{26}{7}=$
14. $\frac{13}{2}=$
15. $\frac{16}{4}=$
16. $\frac{27}{12}=$
17. $\frac{21}{8}=$
18. $\frac{25}{3}=$
19. $\frac{23}{6}=$
20. $\frac{29}{24}=$

Exercise 64

Express the sums in their lowest terms:

1. $\frac{4}{9}+\frac{3}{9}=$
2. $\frac{3}{4}+\frac{2}{4}=$
3. $\frac{1}{3}+\frac{2}{3}=$
4. $\frac{7}{9}+\frac{5}{9}=$
5. $\frac{5}{7}+\frac{6}{7}=$
6. $\frac{13}{18}+\frac{11}{18}=$
7. $\frac{4}{5}+\frac{3}{5}=$
8. $\frac{2}{7}+\frac{3}{7}=$
9. $\frac{4}{5}+\frac{3}{5}=$
10. $\frac{5}{8}+\frac{2}{8}=$
11. $\frac{11}{12}+\frac{9}{12}=$
12. $\frac{2}{3}+\frac{2}{3}=$
13. $\frac{7}{8}+\frac{5}{8}=$
14. $\frac{8}{9}+\frac{4}{9}=$
15. $\frac{9}{10}+\frac{6}{10}=$
16. $\frac{3}{4}+\frac{2}{4}=$

Exercise 65

Express the remainders in their lowest terms:

1. $\frac{7}{8}-\frac{3}{8}=$
2. $\frac{8}{9}-\frac{4}{9}=$
3. $\frac{4}{5}-\frac{2}{5}=$
4. $\frac{5}{9}-\frac{3}{9}=$
5. $\frac{5}{6}-\frac{3}{6}=$
6. $\frac{7}{8}-\frac{5}{8}=$
7. $\frac{9}{12}-\frac{7}{12}=$
8. $\frac{4}{7}-\frac{3}{7}=$
9. $\frac{15}{16}-\frac{11}{16}=$
10. $\frac{9}{10}-\frac{6}{10}=$
11. $\frac{8}{9}-\frac{3}{9}=$
12. $\frac{5}{8}-\frac{3}{8}=$
13. $\frac{9}{11}-\frac{3}{11}=$
14. $\frac{21}{24}-\frac{13}{24}=$
15. $\frac{3}{4}-\frac{1}{4}=$
16. $\frac{5}{7}-\frac{2}{7}=$

Exercise 66

Give the *least* common denominator for each pair of fractions:

1. $\frac{1}{2}, \frac{1}{3}$
2. $\frac{3}{8}, \frac{5}{6}$
3. $\frac{1}{4}, \frac{3}{5}$
4. $\frac{4}{5}, \frac{5}{7}$
5. $\frac{5}{8}, \frac{3}{5}$
6. $\frac{1}{3}, \frac{11}{14}$
7. $\frac{5}{9}, \frac{1}{6}$
8. $\frac{13}{15}, \frac{5}{6}$
9. $\frac{1}{3}, \frac{3}{4}$
10. $\frac{5}{12}, \frac{3}{8}$
11. $\frac{5}{6}, \frac{11}{21}$
12. $\frac{11}{14}, \frac{3}{4}$
13. $\frac{2}{3}, \frac{4}{7}$
14. $\frac{5}{6}, \frac{3}{4}$
15. $\frac{3}{4}, \frac{7}{8}$
16. $\frac{7}{12}, \frac{5}{9}$

Exercise 67

Give the *least* common denominator for each pair of fractions:

1. $\frac{7}{8}, \frac{7}{10}$
2. $\frac{2}{3}, \frac{4}{5}$
3. $\frac{1}{6}, \frac{3}{7}$
4. $\frac{5}{8}, \frac{3}{14}$
5. $\frac{1}{7}, \frac{1}{9}$
6. $\frac{3}{4}, \frac{5}{7}$
7. $\frac{5}{21}, \frac{1}{7}$
8. $\frac{2}{5}, \frac{5}{9}$
9. $\frac{3}{4}, \frac{5}{18}$
10. $\frac{1}{2}, \frac{3}{9}$
11. $\frac{1}{4}, \frac{7}{9}$
12. $\frac{1}{4}, \frac{7}{10}$
13. $\frac{4}{5}, \frac{3}{10}$
14. $\frac{2}{3}, \frac{7}{11}$
15. $\frac{1}{2}, \frac{5}{7}$
16. $\frac{3}{8}, \frac{2}{3}$

Exercise 68

Express the sums in their lowest terms:

1. $\frac{1}{2}+\frac{1}{4}=$
2. $\frac{2}{3}+\frac{5}{6}=$
3. $\frac{1}{3}+\frac{5}{9}=$
4. $\frac{1}{3}+\frac{5}{6}=$
5. $\frac{1}{2}+\frac{3}{4}=$
6. $\frac{3}{4}+\frac{5}{12}=$
7. $\frac{1}{4}+\frac{5}{8}=$
8. $\frac{4}{5}+\frac{9}{10}=$
9. $\frac{1}{4}+\frac{3}{16}=$
10. $\frac{1}{5}+\frac{7}{10}=$
11. $\frac{3}{4}+\frac{5}{8}=$
12. $\frac{3}{8}+\frac{11}{16}=$

The pupil should do the work mentally and merely set down the answers in this exercise.

Exercise 69

Express the sums in their lowest terms:

1. $\frac{2}{3}+\frac{3}{4}=$
2. $\frac{5}{6}+\frac{11}{21}=$
3. $\frac{3}{8}+\frac{5}{6}=$
4. $\frac{13}{15}+\frac{5}{6}=$

Exercise 70

Express the sums in their lowest terms:

1. $\frac{3}{4}+\frac{5}{6}=$
2. $\frac{3}{4}+\frac{5}{18}=$
3. $\frac{2}{3}+\frac{7}{8}=$
4. $\frac{3}{4}+\frac{11}{14}=$

Exercise 71

Express the remainders in their lowest terms:

1. $\frac{5}{6}-\frac{1}{2}$
2. $\frac{11}{12}-\frac{5}{6}=$
3. $\frac{4}{6}-\frac{1}{3}=$
4. $\frac{8}{9}-\frac{2}{3}=$
5. $\frac{11}{16}-\frac{3}{8}=$
6. $\frac{9}{10}-\frac{3}{5}=$
7. $\frac{5}{9}-\frac{1}{3}$
8. $\frac{4}{6}-\frac{1}{2}=$
9. $\frac{5}{8}-\frac{1}{4}=$
10. $\frac{3}{4}-\frac{1}{2}=$
11. $\frac{7}{8}-\frac{3}{4}=$
12. $\frac{7}{10}-\frac{1}{2}=$

Exercise 72

Express the remainders in their lowest terms:

1. $\frac{7}{8}-\frac{2}{3}=$
2. $\frac{5}{6}-\frac{3}{8}=$
3. $\frac{3}{4}-\frac{3}{5}=$
4. $\frac{5}{6}-\frac{3}{14}=$

Exercise 73

Express the remainders in their lowest terms:

1. $\frac{7}{9}-\frac{1}{4}=$
2. $\frac{11}{14}-\frac{3}{4}=$
3. $\frac{11}{12}-\frac{5}{9}=$
4. $\frac{5}{8}-\frac{3}{5}=$

Additional exercises may be made by using 2 examples from exercise 69 or 70 and 2 examples from exercise 72 or 73.

Exercise 74

Express the products in lowest terms:

1. $\frac{3}{4} \times \frac{16}{21} =$
2. $\frac{7}{12} \times \frac{3}{4} =$
3. $\frac{19}{24} \times \frac{4}{5} =$
4. $\frac{3}{4} \times \frac{3}{5} =$
5. $\frac{3}{5} \times \frac{7}{9} =$
6. $\frac{2}{3} \times \frac{3}{4} =$
7. $\frac{5}{8} \times \frac{16}{20} =$
8. $\frac{11}{3} \times \frac{9}{4} =$
9. $\frac{4}{5} \times \frac{5}{8} =$

Exercise 75

Express the quotients in their lowest terms:

1. $\frac{3}{4} \div \frac{5}{8} =$
2. $\frac{8}{9} \div \frac{4}{6} =$
3. $\frac{7}{12} \div \frac{3}{4} =$
4. $\frac{15}{16} \div \frac{3}{4} =$
5. $\frac{7}{8} \div \frac{5}{4} =$
6. $\frac{2}{3} \div \frac{4}{5} =$
7. $\frac{3}{4} \div \frac{3}{5} =$

Exercise 76

Add:

1.	2.	3.	4.	5.	6.
$4\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$7\frac{1}{2}$	$2\frac{1}{2}$	$6\frac{3}{5}$
$\underline{2\frac{3}{4}}$	$\underline{3\frac{5}{8}}$	$\underline{2\frac{5}{6}}$	$\underline{3\frac{7}{8}}$	$\underline{3\frac{1}{3}}$	$\underline{2\frac{2}{3}}$

Exercise 77

Subtract:

1.	2.	3.	4.	5.	6.
$5\frac{1}{2}$	$4\frac{1}{2}$	$12\frac{5}{6}$	$9\frac{5}{8}$	$12\frac{1}{3}$	$6\frac{3}{4}$
$\underline{2\frac{2}{3}}$	$\underline{1\frac{3}{4}}$	$\underline{4\frac{1}{3}}$	$\underline{6\frac{1}{4}}$	$\underline{5\frac{5}{6}}$	$\underline{5\frac{2}{3}}$

When these exercises are hectographed or copied, sufficient space should be allowed between them for all the work to be performed.

The teacher should devise other drill exercises of the same length and the same difficulty to provide additional practice material.

Exercise 78

Multiply:

1. $3\frac{2}{3}\times5\frac{1}{4}=$
2. $5\frac{1}{2}\times5\frac{1}{2}=$
3. $9\frac{1}{4}\times1\frac{1}{2}=$
4. $7\frac{1}{2}\times8\frac{2}{5}=$

Exercise 79

Divide:

1. $5\frac{3}{4}\div1\frac{7}{8}=$
2. $3\frac{1}{2}\div2\frac{3}{4}=$
3. $2\frac{2}{3}\div1\frac{3}{5}=$

Exercise 80. Review Problems in Fractions

1. A carpenter worked $16\frac{1}{5}$ hours at 65 cents an hour. How much did he receive for this work?

2. Find the cost of $9\frac{1}{2}$ pounds of nails at $7\frac{1}{2}$ cents a pound.

3. Paul planted his onions in rows $1\frac{1}{4}$ feet apart. How many rows did he have if his plot was 30 feet wide?

4. How many cabbage plants $1\frac{1}{2}$ feet apart can be set in a row 96 feet long?

5. A Camp-Fire girl bought $6\frac{1}{2}$ pounds of sugar; $4\frac{3}{4}$ pounds of bacon; 5 pounds of corn meal and $3\frac{1}{3}$ pounds of frankfurts. How many pounds of supplies did she purchase?

6. A chum who was with her asked to carry half of the supplies. How could they divide the 4 packages so that they would have about equal weights to carry?

7. Joe caught a catfish that weighed $1\frac{3}{4}$ pounds and Tom caught a carp that weighed $2\frac{1}{8}$ pounds. How much heavier was Tom's fish than Joe's?

8. On one fishing trip I caught 24 fish which weighed 14 pounds. What was the average weight of these fish?

9. I sold 8 fish which averaged $\frac{3}{4}$ of a pound each in weight. How many pounds did I have left?

10. Three girls made $11\frac{1}{2}$ pounds of candy to sell at a social. They sold $\frac{1}{4}$ of a pound for 10 cents. How much did they receive for the candy if they disposed of all of it at that rate?

11. In a track meet the best running jump made by the sixth grade was $13\frac{1}{4}$ feet. The best fifth-grade jump was $12\frac{1}{3}$ feet. How much farther did the sixth-grade boy jump?

12. Mae bought 28 inches of 44-inch oilcloth to cover the kitchen cabinet. How much did it cost at 24 cents a yard?

13. Find the cost of $1\frac{3}{4}$ pounds of round steak at 28 cents a pound.

14. In 1915 tin cans cost $2\frac{1}{2}$ cents each. In 1916 they had advanced to 3 cents each. How much did this rise in price increase the expenses of canning 90 cans of tomatoes?

15. A firm paid their office girls $\frac{1}{5}$ cent per letter extra for each letter beyond 100 per hour. How much extra did a girl make per hour who read 140 letters per hour?

16. If a hog gains $11\frac{1}{2}$ pounds for every bushel of corn that he eats, how many bushels, at that rate, will be required to increase his weight $80\frac{1}{2}$ pounds?

17. Find the cost of $5\frac{1}{4}$ yards of linen at $\$\frac{3}{5}$ per yard.

18. From $11\frac{1}{2}$ pounds of butter, $3\frac{3}{4}$ pounds were sold to one customer and $3\frac{7}{8}$ pounds to another. How much was left?

19. A farmer bought 15 bushels of oats. If he feeds each of his two horses $\frac{3}{8}$ of a bushel per day, how many days will the oats last?

20. A squad of Boy Scouts walked $8\frac{3}{4}$ miles in the morning and $7\frac{5}{8}$ miles in the afternoon. How far did they walk during the day?

21. The items on my laundry bill one week were 6 shirts at $12\frac{1}{2}$ cents each and 8 collars at $2\frac{1}{2}$ cents each. Find the total charge for those items.

22. A class divided 12 yards of ribbon into badges. How long did they cut each badge to provide one for each of the 32 pupils in the class?

23. George made a bread board by gluing three boards $3\frac{1}{2}$ inches, $2\frac{3}{8}$ inches and $2\frac{5}{16}$ inches wide together. How wide was the bread board?

24. Mrs. Anderson ordered a pail of butter from a farmer in the country. The butter and pail weighed $11\frac{1}{2}$ pounds. The pail weighed $\frac{3}{4}$ of a pound. What was the weight of the butter?

25. How much did this butter cost her at 35 cents a pound? The farmer charged 15 cents for the pail, and the express charges were 26 cents. What was the total cost of the pail of butter?

26. Find the cost per pound of the butter delivered.

27. Mrs. Anderson divided the pail of butter with a neighbor. She sold her $5\frac{3}{16}$ pounds. How much did she receive for this amount, charging for it at the price delivered?

28. It is estimated by dry-goods stores that at least $\frac{1}{2}$ yard of cloth is wasted by crooked cutting in every bolt. How much is the value of this waste in cloth worth $1.50 a yard?

29. How many guest towels $\frac{3}{4}$ of a yard long can be made from a piece of toweling $3\frac{3}{4}$ yards long?

30. A sewing teacher bought 24 yards of percale for a class to make aprons. It took $1\frac{1}{2}$ yards to make an apron. How many aprons could be made from that amount?

31. Find the cost of the following meat bill: $1\frac{1}{2}$ pounds of steak at 28 cents a pound; a $5\frac{1}{4}$-pound roast at 22 cents a pound; $1\frac{3}{4}$ pounds of whitefish at 18 cents a pound and $1\frac{5}{8}$ pounds of calve's liver at 25 cents a pound.

32. A bushel of wheat occupies $1\frac{1}{4}$ cubic feet of space. How many cubic feet of space are required for 120 bushels of wheat?

Exercise 81. Dressmaking Problems

1. A piece of goods 16 inches long contains 4 quarter-inch tucks, 3 half-inch tucks and a two-inch hem. How long was it before being tucked?

Allow double for tucks. For example: A quarter-inch tuck requires one-half of an inch of material. Allow $\frac{1}{4}$ inch extra for all hems to allow for turning under the edge.

2. How wide must a piece of goods be to be 24 inches wide when complete if we allow for 6 quarter-inch tucks and a 2-inch hem?

3. Find the cost of the following serge dress: 4 yards of serge at $1.00 a yard; $1\frac{1}{2}$ yards of braid at 18 cents a yard; 1 yard of lining at 12 cents; 1 yard stiffening at 10 cents; thread 15 cents; 1 card of hooks and eyes 10 cents and 1 bolt bias binding 5 cents.

4. Find the cost of a school apron: $1\frac{1}{2}$ yards percale at $12\frac{1}{2}$ cents a yard; 1 bolt ric-rac braid at 5 cents; 2 buttons at 10 cents a dozen; $\frac{1}{4}$ spool of thread at 5 cents.

5. A girl in the sewing class made a serge dress. She bought $3\frac{1}{2}$ yards of 54-inch serge at $1.00 a yard; $\frac{1}{2}$ yard of lining at $.25 a yard; 1 card of hooks and eyes 10 cents; 1 bolt of bias binding at 10 cents; 1 bolt of braid at 18 cents; 1 spool of silk thread at 10 cents; 1 cotton spool 5 cents and 10 cents worth of belting. Find the cost of her dress.

6. Find the cost of the materials for a gingham school dress: $8\frac{1}{2}$ yards of gingham at $12\frac{1}{2}$ cents a yard; $\frac{1}{2}$ yard linen at 30 cents a yard; 1 bolt bias binding at 10 cents; 1 card of hooks and eyes at 5 cents; 1 spool of thread at 5 cents and 1 yard belting at 10 cents.

7. Find the width of a piece of goods if it is to be 20 inches wide when finished if we are to allow for four $\frac{1}{2}$-inch tucks, six $\frac{1}{8}$-inch tucks, two $\frac{1}{4}$-inch tucks and a 1-inch hem.

Review of Decimals

What are the chief points to observe in copying decimals for addition or subtraction?

Exercise 82

Copy in column form and add:

1. 2.073; .0625; 1.02; 16.5.
2. 50.3; 1.875; .0075; 3.05.
3. .625; 21.07; 8.003; 2.5.
4. 123.5; 6.25; .043; 6.06.
5. 1.003; 60.2; .081; 2.32.
6. 2.001; 30.06; 1.008; .015.
7. .0025; 10.031; 3.01; 321.4.
8. 5.25; 20.05; .0005; 62.5.
9. 32.05; 1.25; 6.025; 1.4142.
10. .035; 6.025; 11.25; 1.0075.

Exercise 83

Copy in column form and subtract:

1. 5.025 − 3.75.
2. 24.005 − 7.875.
3. 1.9205 − .8762.
4. 62.5 − 3.1416.
5. 2150.42 − 1728.
6. 20.75 − 14.625.
7. 4.003 − 2.126.
8. 3.5 − 1.4142.
9. 2.45 − 1.725.
10. 5.625 − 2.5.
11. 11.075 − 8.75.
12. 3.0 − 2.45.
13. 127.15 − 65.4.
14. .85 − .625.
15. 13.5 − 6.25.
16. 34.25 − 1.075.

Exercise 84

State the principle for pointing off the decimal places in a product.

Multiply:

1.	**2.**	**3.**	**4.**	**5.**
$3.50	$1.46	$10.50	375 lb.	$151.75
28.	21.5	.75	.038	.06
6.	**7.**	**8.**	**9.**	**10.**
62.5 lb.	3.1416	.866	2150.42	$4.50
.82	7.5	6.75	23.75	1.02
11.	**12.**	**13.**	**14.**	**15.**
1.005	.0025	1.532	$62.75	83.5
.065	.075	.65	.045	.039
16.	**17.**	**18.**	**19.**	**20.**
3.1416	1.4142	376	1.15	1.732
4.25	8.5	.05	18.5	1.732

Exercise 85

State the principle for pointing off decimal places in a quotient.

Divide (carry the quotients to four decimal places if necessary):

1. .1088 by .032.

2. .025 by 12.5.

3. 21.9912 by 3.1416.

4. 10.392 by .866.

5. 78.54 by 25.

6. 62.5 by .125.

7. $78.60 by .05.

8. $13.75 by .055.

9. .0075 by 2.5.

10. 3.75 by .0125.

11. 5 by 80.
12. 804.44 by 36.4.
13. .07812 by .28.
14. .025 by .0005.
15. 21.411 by 7.02.
16. 31.5 by .375.
17. 9.1923 by 1.4142.
18. 144.375 by 16.5.
19. 484 by 30.25.
20. 111 by 15.
21. 18.792 by 3.48.
22. 337.68 by 67.
23. $12.25 by $175.
24. 50534.87 by 2150.42.

Exercise 86. Applied Problems in Decimals

1. A barrel holds 31.5 gallons. How many bottles holding .125 of a gallon can be filled from a barrel of vinegar?

2. A knot is the unit of measure in estimating the speed of ships. It is equal to 1.15 miles. The battleship Pennsylvania made 21.05 knots per hour in its trial speed test. Express the speed of this battleship in miles per hour.

3. The torpedo boat destroyer Jouett made 32.27 knots per hour in its trial speed test. How many miles per hour is that record?

4. A passenger train, on a 180.5-mile run, made an average speed of 42.5 miles per hour. How long did it take the train to make the run?

5. At that rate how far would the train go in 2.25 hours?

6. A touring car was driven 82.5 miles on 5 gallons of gasoline. What was the average distance obtained from a gallon of gasoline?

7. A bushel is equal to 1.25 cubic feet. How many bushels will a bin hold whose volume is 162.5 cubic feet?

8. The railroad fare between two cities is $2.46. The rate between these cities is 2.4 cents a mile. Find the distance between the cities.

9. A square rod is equal to 30.25 square yards. How many square rods are there in a plot of ground containing 931.5 square yards?

10. A rod = $16\frac{1}{2}$ feet. How many rods long is a garden plot 330 feet long?

11. Jack sold a grocer 12 cabbages which weighed 28.5 pounds. What was the average weight of the cabbage?

12. A cubic foot of water weighs 62.5 pounds. Ice weighs only .92 as much as water. How much does a cubic foot of ice weigh?

13. Gasoline weighs only .76 as much as water. How much does a cubic foot of gasoline weigh? There are 7.5 gallons in a cubic foot. How much does a gallon of gasoline weigh?

14. The distance from New York to Chicago is 952 miles. How long will it take a train to go from New York to Chicago at an average speed of 36.75 miles per hour?

15. The record for the 100-yard dash for a high-school meet was 10.8 seconds. The record in a grammar-school meet was 12.2 seconds. How many seconds shorter was the high-school record?

16. The English shilling is worth $.243375 in our money. How much is 8 shillings worth in our money?

17. A cubic foot of water weighs 62.5 pounds. How much does a barrel, 4.2 cubic feet, of water weigh?

18. A cubic foot of iron weighs about 465 pounds. How many cubic feet of iron weigh a ton?

19. A rectangular field is 24.75 rods long and 15.625 rods wide. How many rods of wire fence will it take to enclose the field?

20. Find the length of the four sides of a rectangle that is 7.5 inches long and 5.625 inches wide.

Exercise 87. Selling Goods

1. A certain grade of toweling sells at 50 cents a yard. How much would you charge a customer for $\frac{3}{4}$ of a yard?

2. How much should be charged for $\frac{3}{4}$ of a yard of veiling at 75 cents a yard?

3. Handkerchiefs of a certain quality sell at 3 for 25 cents. How much should you charge for $1\frac{1}{2}$ dozen?

4. Find the total for $2\frac{1}{2}$ dozen eggs at 27 cents a dozen.

5. If Dutch Cleanser sells at 10 cents each or 3 for 25 cents, give two reasons why you would suggest to your customer the purchase of 3 rather than one.

6. Find the total for the following bill: 2 bottles of Maraschino cherries at 10 cents each; 25 pounds of sugar at 6 cents; 6 cans of tomatoes at 15 cents; 1 package of salt at 9 cents; 3 packages of macaroni at 30 cents, and 12 bars of soap at 48 cents.

7. If sugar is 8 cents per pound, how many pounds do you sell for $1.00; for 50 cents; for $1.50; for 25 cents?

8. Tomatoes are selling at $12\frac{1}{2}$ cents a can. How many cans do you sell for 50 cents; for 25 cents; for 75 cents?

9. What are the charges for $2\frac{1}{2}$ yards of sheeting at 28 cents a yard?

10. How much would you charge for 18 yards of toweling at $12\frac{1}{2}$ cents a yard?

11. Bring to the class any sales slips that you can find at home and dictate the items to the class for rapid work in adding up sales slips.

Wireless

The Navy Department makes extensive use of wireless for giving orders to the vessels of our navy when they are at sea.

The central station of the United States Naval Radio Service is stationed at Radio, Virginia, and is generally called the Arlington Radio Station.

The Arlington station has three skeleton towers, one 600 feet high and two each 450 feet high. Five hundred tons of steel were used in the construction of the highest tower and 275 tons in each of the smaller towers. Why is it necessary to have such high towers to send wireless messages?

Exercise 88

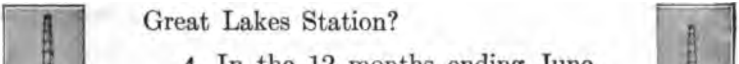
1. How many tons of steel were used in all of the Arlington towers?

2. How much did the steel cost for the three towers at $56.20 per ton?

3. The distance from Arlington to the wireless station in Darien is 1779 miles. The distance to the Great Lakes Station (shown in the illustration) is 1000 miles. How much farther is it to Darien than to the Great Lakes Station?

4. In the 12 months ending June 30, 1915, Arlington handled 78,921 messages. Find the average number of messages handled per day.

5. In January, 1916, the Arlington Station handled 6189 messages. What was the average per day during that month? How many messages did this station send per hour?



The Great Lakes Wireless Station

Exercise 89

There are 4 wireless telegraph systems in the United States having 74 tower stations distributed over the country.

1. In 1916 these 74 stations sent 285,091 messages. What was the average number of messages sent during that year by each station?

2. What was the average number of messages sent by each station per month?

3. The total number of land telegraph offices in 1916 was 30,781. The number of messages sent by these offices was 103,536,418. Find the average number of messages sent by each of these offices per month.

4. The total income from the 285,091 messages sent by wireless was $669,158. What was the average cost of sending a wireless message?

5. The total income from the 103,536,418 land messages was $52,293,469. What was the average cost of sending a land telegraph message?

6. The income of the wireless systems was $669,158 and the expenses were $664,420. What was the net income from the wireless messages? What was the net profit on each message?

(See problem 1 for number of messages.)

7. The average number of employees of the wireless systems in 1916 was 958. The salaries and wages of these employees amounted to $393,606. What was the average amount received by each employee?

Pupils should be encouraged to bring to class any data on the cost of setting up a simple wireless plant. In almost every class you will find one or more boys interested in wireless and their information should be given to the class and made the base for a lesson in arithmetic. See page 100 for the cost of a simple Wireless system recommended in the "Boy Scouts of America."

Exercise 90. Selling School Supplies[1]

The class of children who sold bulbs also had a store in which they sold most of their school supplies that were not furnished. A long table for the counter, a pair of scales for weighing paper, a change box and spindles which the boys made were all the furnishings of the store.

The stock consisted of pencils, rulers, penholders, erasers, spelling blanks, tablets, scratch pads, paint boxes, extra brushes, new pans of colors, boxes of crayons, loose-leaf notebook covers and notebook paper. All of these supplies except the notebook covers and paper were ordered from the local dealers.

Find the amount of each of the items in their first order:

1.	5 dozen Old Glory tablets	@	$.05 each
2.	3 dozen spelling blanks	@	.05 each
3.	$1\frac{1}{2}$ dozen paint boxes	@	.25 each
4.	36 extra color pans	@ 2 for	.05
5.	$\frac{1}{2}$ dozen pans of gold paint	@	.10 each
6.	2 dozen paint brushes	@	.03 each
7.	2 dozen ink and pencil erasers	@	.05 each
8.	1 dozen pencil erasers	@	.05 each
9.	2 dozen pencils	@	.01 each
10.	2 dozen pencils	@ 2 for	.05

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[1] The teacher will find it much better to organize a store on this plan and let the arithmetic be based on the real activity. In such a situation this section of the book can be used by the pupils for suggestions as to how to carry on their business.

11. 1 dozen pencils.............................@ .05 each

12. 18 cork-tipped penholders...................@ .05 each

13. 18 rubber-tipped penholders.................@ .05 each

14. 1 dozen boxes of crayons....................@ .15 each

15. 36 scratch pads............................@ .01 each

16. 3 dozen rulers.............................@ .05 each

17. Find the total amount of their first order of supplies.

Exercise 91. Ordering Goods

1. Imagine that you are the Fifth Grade and write a business letter to a book store, giving the following order: 20 loose-leaf notebook covers at $0.20 a pair; 8 reams of white notebook paper at $0.85 a ream and 4 reams of yellow paper at $0.40 a ream.

If you do not know the proper form of a business letter, look it up in your grammar or some book on English composition.

2. The class sent a postal money order to pay for these supplies. From the following rates find the amount paid for the money order.

In addition to the amount of the money order the post office charges a fee for the various amounts as follows:

To send orders from $ 0.01 to $ 2.50 costs 3 cents extra.
To send orders from 2.51 to 5.00 costs 5 cents extra.
To send orders from 5.01 to 10.00 costs 8 cents extra.
To send orders from 10.00 to 20.00 costs 10 cents extra.

3. Another class also organized a store. They sent in the following order for supplies: 15 notebook covers at $0.20; 2 reams of white notebook paper at $0.85 and 1 ream of yellow paper at $0.40. Write a business letter ordering these supplies.

4. Find the total cost of a money order to pay for the supplies ordered in Problem 3.

Exercise 92. Selling Goods

The store opened for business Wednesday, September 6, although the notebook covers and paper did not arrive until Friday. A record of each sale was made on a sales slip like the one shown on page 105. The sales slip was given to the cashier when the goods were paid for, and he placed it on a spindle to keep for figuring up the accounts of the business.

The following sales[1] were made September 6. Find the amount of each sales slip.

1. One spelling blank, 1 paint box, 1 ruler, 1 five-cent pencil, 2 one-cent pencils, 1 cork-tipped penholder, 1 Old Glory tablet.

2. One ruler, 1 box crayons, 2 pans of color, 1 brush, 2 scratch pads, 2 two-for-five-cent pencils, 1 Old Glory tablet, 1 spelling blank.

3. One paint box, 1 spelling blank, 1 Old Glory tablet, 1 rubber-tipped penholder, 1 scratch pad, 1 ink and pencil eraser, 5 one-cent pencils.

4. One spelling blank, 1 five-cent pencil, 1 cork-tipped penholder, 2 pans of color, 3 scratch pads, 1 Old Glory tablet, 1 ruler.

5. One ruler, 1 spelling blank, 4 two-for-five-cent pencils, 1 Old Glory tablet, 2 pans of color, 1 pan of gold paint.

6. Two rulers, 3 spelling blanks, 8 two-for-five-cent pencils, 2 pencil erasers, 2 cork-tipped penholders, 1 box crayons, 2 boxes of paints.

7. Three Old Glory tablets, 4 scratch pads, 1 rubber-tipped penholder, 2 five-cent pencils, 4 pans of color, 2 spelling blanks.

8. One tablet, 2 spelling blanks, 1 scratch pad, 4 one-cent pencils, 2 rulers.

9. Find the total amount of the first day's sales.

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[1] Since there was no expense connected with the supplies secured through the local dealer, it was decided to sell these supplies at cost.

Exercise 93. Taking an Inventory

It is the custom in most stores to take an inventory at least once a year. To take an inventory is to count and make a record of all unsold stock and state its value based on its cost. The class took an inventory of their stock September 8.

Inventory, September 8

1. Paper and tablets:				
4 reams white paper	@	$.85=	?	
2 reams yellow paper	@	.40=	?	
21 scratch pads	@	.01=	?	
8 spelling blanks	@	.05=	?	
26 "Old Glory" tablets	@	.05=	?	
5 notebook covers	@	.20=	?	
2. Pencils:				?
8	@	$.01=	?	
1	@	.05=	?	
3. Penholders:				?
13 cork-tipped	@	$.05=	?	
13 rubber-tipped	@	.05=	?	
4. Erasers:				?
20 ink and pencil	@	$.05=	?	
10 pencil	@	.05=	?	
5. Art supplies:				?
10 paint boxes	@	$.25=	?	
22 color pans	2 for	.05=	?	
3 pans of gold paint	@	.10=	?	
17 paint brushes	@	.03=	?	
9 boxes of crayons	@	.15=	?	
6. Rulers:				?
16	@	$.05=	?	?
Total amount on hand				?

7. How many divisions of stock are shown?

8. Find the value for each item and also for each division of the inventory.

9. Add the column of totals for each division and find the total value of all the stock listed.

10. Prepare an inventory of the school supplies in your room. Use a school supply catalogue to get the prices of the various articles.

Exercise 94. Providing for the Extra Expenses

Since the class had to pay express and a money-order fee in securing the notebook covers and paper, it was decided to sell the paper at a price slightly above the cost in order to provide for these extra expenses.

1. A ream of white paper weighed 6 pounds and cost \$.85. Most of it was sold at the rate of $\frac{1}{2}$ pound for \$.08. What was the selling price of a pound?

2. At this rate what was the selling price of a ream?

3. Find the gain on 4 reams sold in this way.

4. The yellow paper weighed 3 pounds to the ream and cost \$.40 per ream. Find the gain on 1 ream sold at \$.15 a pound.

5. How much was gained on 2 reams sold in this way?

6. Find the gain on 4 reams sold in this way.

Exercise 95. The Cash Account

In order to keep a record of the store's business, the class kept a cash account, showing the amounts received and the amounts paid out. This cash account was balanced each week as shown in the form on the next page:

The cashier added up the sales slips to find the total sales for the day and compared that amount with the increase in his cash box to see that he had made no mistakes in giving change.

Cash Account

1916		Received			1916		Paid		
Sept.	6	Sales	5	43	Sept.	8	Express		35
Sept.	7	Sales	5	01	"	"	Check to A.L.Zick	15	00
Sept.	8	Sales	6	98			Balance on hand	2	07
			17	42				17	42
Sept.	11	Cash on hand	2	07					

1. How does the amount of money paid out compare with the amount of money received?

2. How do we find the balance on hand?

In business, the books are usually balanced once a month. This class balanced their cash account each week because they had the store only for the first three weeks of school.

Exercise 96. Repairing Shoes

The pupils of a certain city school repair shoes for the pupils of that school at the following prices per shoe:

Half-soling shoe, 50c.	Sewing half-sole, 60c.
Rubber heels, 40c.	Re-fastening buttons, 5c.

1. Find the cost of half-soling eight pairs of shoes, the soles to be sewed.

2. What will be the cost of half-soling five pairs of shoes (pegged), and putting on rubber heels?

3. Find the cost of putting on rubber heels and re-fastening the buttons on six pairs of shoes.

Some school boards have equipped their manual training departments with shoe-repairing outfits so that pupils unable to pay may have their shoes kept in repair, *during the school months*, free of charge.

CHAPTER II

DENOMINATE NUMBERS

Exercise 1

Review the tables of length, square measure, cubic measure and weight found on pages 129 to 140. Supply the missing numbers in the following:

1 foot = ? inches
1 mile = ? rods
1 ton = ? pounds
1 yard = ? feet
1 mile = ? yards
1 square yard = ? square feet
1 cubic yard = ? cubic feet
1 acre = ? square rods

1 rod = ? yards
1 mile = ? feet
1 pound = ? ounces
1 square foot = ? square inches
1 cubic foot = ? cubic inches
1 square rod = ? square yards
1 rod = ? feet
1 yard = ? inches

1. How many rods are run in a quarter-mile race? How many yards?

2. A-hundred yard dash is equal to how many feet? To how many rods?

3. Find the number of inches in a mile.

4. A field is 40 rods long and 30 rods wide. Find the number of square rods in the field. (See page 129.) How many acres are there in the field?

5. A cellar is 20 feet long, 15 feet wide and 8 feet deep. How many cubic feet are there in the cellar? (See page 136.) How many cubic yards of earth were removed to make the excavation for the cellar if the excavation was 6 feet deep?

6. The average weight of a bale of cotton is 500 pounds. How many such bales does it take to weigh a ton?

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[1] See page 200 for definition of denominate numbers.

Exercise 2. Liquid Measures

You have already had many opportunities to use pints, quarts and gallons in your daily experiences.

1. What sized bottles or measures are usually used in selling milk and cream?

2. What is the chief measure used in selling gasoline and kerosene?

3. In what sized bottles or measures is vinegar sold?

4. What other liquids are measured in pints, quarts or gallons?

Table of Liquid Measure

2 pints[1] (pt.) = 1 quart (qt.)
4 quarts = 1 gallon[2] (gal.)
A gallon = 231 cubic inches
A cup or common tumbler = $\frac{1}{2}$ pint

5. How many quarts of lemonade are needed to serve a party of 24 children, allowing a tumbler full for each?

6. A school lunch room uses 60 quarts of milk per day. How many gallons of milk do they use? How many half-pint bottles can be filled with 60 quarts of milk?

7. How much does this milk cost the lunch room at 10 cents per quart?

8. A recipe for a cake calls for a cup of milk. What part of a quart bottle of milk will be used in making the cake?

---

[1] Formerly 4 gills were considered as a pint, but the gill is now very little used. It has been superseded by the cup or half-pint.

[2] The gallon takes its name from an old French wine jar or bowl.

9. Find the volume in cubic inches of a rectangular shaped watering trough 70 inches long, 12 inches wide and 10 inches deep. How many gallons will it hold?

Remember that a gallon = 231 cubic inches.

10. How many half-pint bottles of grape juice can be filled from a kettle containing 2½ gallons?

Barrels vary greatly in size. A standard barrel holds 31½ gallons.

11. How many quart bottles can be filled from a barrel of vinegar?

12. A hogshead, in this country, generally holds 63 gallons. How many barrels make a hogshead?

13. A pint of water weighs 1.044275 pounds. How much does a gallon of water weigh?

14. A pint[1] of milk weighs 1.07775 pounds. How much does a gallon of milk weigh?

15. Milk is sold in large quantities by weight. How many gallons of milk are there in a can containing 100 pounds of milk?

16. Why is it more convenient to sell milk in large quantities by weight rather than by measure?

Exercise 3. Dry Measures

Such articles as wheat, corn, potatoes, apples, berries, etc., are measured in bushels, pecks, quarts and pints. In selling potatoes and apples, the measures must be heaped to give the right amount. Why?

---

[1] Since a pint of water, milk and certain other liquids weighs approximately a pound, there originated the old saying that:

"A pint's a pound, the world around."

Since it is difficult to tell just how much to heap a measure for each article, it is much more satisfactory to buy and sell these products by weight. Many stores sell potatoes, apples, beans, rice, etc., entirely by weight. They price apples at 10 pounds for 45 cents or 15 pounds of potatoes for a certain amount. Other stores sell by weight, but still use the terms bushels, pecks and quarts. It is therefore necessary to know the facts in the table of dry measure.

Table of Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains 2150.42 cubic inches. A heaped bushel contains 2747.07 cubic inches.

1. A fruit dealer sells crates of strawberries containing 16 quart boxes. How many bushels are there in each crate? How much per bushel is obtained for strawberries at $2.40 a crate?

2. A tin box is 10 inches long, 8 inches wide and 7 inches deep. How many quarts of berries[1] will it hold?

See footnote for number of cubic inches in 1 quart.

3. A barrel of apples holds approximately 3 bushels. How many pecks of apples are there in a barrel? How much does a peck of apples cost at $6.00 a barrel?

4. How many cubic inches are there in a cubic foot? In estimating the capacity of bins, 5 cubic feet are allowed for 4 bushels of grain. How much does the actual number of cubic inches in 4 bushels compare with the number of cubic inches in 5 cubic feet?

---

[1] The quarts and pints are not the same in liquid and dry measures. The liquid quart contains 57¾ cubic inches, and the quart in dry measure contains about 67⅕ cubic inches.

5. A fruit dealer sold crates of blackberries containing 16 quart boxes at $2.20 per crate. Single boxes sold at 15 cents each. How much more would a peck cost when bought by the box than by the crate?

6. A bin is 12 feet long, 8 feet wide and 6 feet high. How many bushels of grain will it hold?

7. How many pint boxes will a bushel of berries fill?

8. If berries are selling at 2 boxes for 25 cents, how much is the cost per peck?

9. A grocer bought 40 bushels of potatoes at 80 cents per bushel and sold them to his customers at 25 cents per peck. How much did he make per peck? How much was his profit on the entire amount at that rate?

10. A farmer bought 30 bushels of oats and fed his three horses 2 quarts each, twice a day. How many days will the 30 bushels of oats last at that rate?

11. A bushel of wheat weighs 60 pounds. Find the weight of a load of 24 sacks containing 2 bushels each.

12. How much profit is made by buying onion sets at $3.20 per bushel and retailing them at 15 cents a quart?

13. How much profit is made per bushel by buying potatoes at 90 cents a bushel and selling them at 30 cents a peck?

14. A fruit peddler buys cherries for $2.00 a bushel and retails them to his customers at 10 cents a quart. What is his profit on each quart?

Most of the states have passed laws stating the number of pounds in a bushel of each of the different products sold by dry measure. Since most stores now sell these products by weight, we should know the legal weights for each of them in our own state.

Legal Weights of a Bushel, in Pounds, for Various Products

STATES	WHEAT	RYE	BUCKWHEAT	SHELLED CORN	OATS	BARLEY	APPLES	POTATOES	SWEET POTATOES	BEANS	PEAS	BEETS	ONIONS	FLAXSEED
Alabama	60	56		56	32	47			55	60	60			
Arizona	60	56			32	45				55				
Arkansas	60	56	52	56	32	48	50	60	50	60	60		57	56
California	60	54	40		32	50								
Colorado	60	56	52		32	48		60		60			57	
Connecticut	60	56	48		32	48	48	60	54	60	60	60	52	55
Delaware	60													
Florida	60	56		56	32	48	48		60	60			56	
Georgia	60	56	52	56	32	47			55	60	60		57	56
Idaho	60	56	42		36	48	45	60						56
Illinois	60	56	52	56	32	48			50	60			57	56
Indiana	60	56	50	56	32	48		60	55	60			48	
Iowa	60	56	52	56	32	48	48	60	46	60			57	56
Kansas	60	56	50		32	48	48	60	50	60			57	56
Kentucky	60	56	56	56	32	47		60	55	60	60		57	56
Louisiana	60	56												
Maine	60	50	48		32	48	44	60		60	60	60	52	
Maryland					26			56						
Massachusetts	60	56	48	50	32	48	48	60	54	60	60		52	55
Michigan	60	56	48	56	32	48	48		56	60	60		54	56
Minnesota	60	56	50	56	32	48	50		55	60	60	50	52	
Mississippi	60	56	48	56	32	48			60	60	60		57	56
Missouri	60	56	52	56	32	48	48		56	60	60		57	56
Montana	60	56	52	56	32	48	45	60		60	60	50	57	56
Nebraska	60	56	52	56	32	48			50	60	60		57	56
Nevada														
N. Hampshire	60	56			32			60		62	60			
New Jersey	60	56	50		30	48	50		54	60	60		57	55
New Mexico														
New York	60	56	48		32	48	48		54	60	60		57	55
North Carolina	60	56	50		32	48					60			55
North Dakota	60	56	42	56	32	48	50		46	60	60	60	52	56
Ohio	60	56	50	56	32	48	50		50	60	60	56	55	56
Oklahoma	60	56	42	56	32	48			46	60	60	60	52	56
Oregon	60	56	42		32	43	45	60						
Pennsylvania	60	56	48		32	47		56					50	
Rhode Island	60	56	48	56	32	48	48		54	60	60	50	50	56
South Carolina														
South Dakota	60	56	42	56	32	48			46	60	60	60	52	56
Tennessee	60	56	50	56	32	48	50		50	60	60	50	56	56
Texas	60	56	42	56	32	48	45		55	60			57	56
Utah														
Vermont	60	56	48		32	48	46	60		62	60	60	52	
Virginia	60	56	52	56	30	48			56	60	60		57	56
Washington	60	56	42		32	48	45	60						56
West Virginia	60	56	52		32	48		60		60				56
Wisconsin	60	56	50		32	48	50		54	60	60	50	57	56
Wyoming														
U. S. Customs	60	56	42		32	48	60	60			60			56
Dist. of Col.								60						

This reference table should be used in connection with the problems on the next page. Every pupil should become familiar with the legal weights of a bushel in his own state.

Exercise 4

1. What is the weight of a bushel of potatoes in your state? How many pounds should you receive for a peck?

2. A bushel of apples weighs how many pounds in your state?

3. A truck farmer sold 2592 pounds of onions in Michigan at 90 cents a bushel. How much did he receive for them?

4. How much would the farmer have received for the same weight of onions in Indiana at 85 cents per bushel?

5. Find the weights of a bushel of the following products in your state: Rye, buckwheat, oats, barley, sweet potatoes, beans, peas and flaxseed.

6. What is the legal weight of a bushel of shelled corn in Massachusetts? How much does a bushel of shelled corn weigh in the other states?

7. A bushel of corn on the cob is usually considered to weigh 70 pounds. Find the weight of the cobs in a bushel of corn on the cob

8. A wholesale dealer bought a carload of sweet potatoes in New Jersey at $1.35 per bushel. Find the cost per pound.

9. If he sold them in Cleveland, Ohio, at $1.50 a bushel, what was his increase in the price per pound on the sweet potatoes? (See the table on page 194).

10. In measuring corn on the ear in cribs, 4000 cubic inches are usually considered a bushel. How many bushels are there in a crib 8 feet long, 8 feet wide and 9 feet high?

11. I bought a peck of apples from a peddler. He used a peck measure in measuring them. I weighed the apples and found that they weighed only 9 pounds. In your state, how much would that amount have been below the standard weight per peck?

Exercise 5. Counting

What articles are sold by the dozen? Name other articles that are sold by the gross.

12 things = 1 dozen
12 dozen = 1 gross

1. How many things are there in a gross?

2. A case of eggs holds 360 eggs. How many dozen eggs are there in a case?

3. Oranges are quoted in a certain fruit store at 39 cents per dozen. How much should the dealer charge you at that rate for a half-dozen? For 3 oranges?

4. Screws are quoted at 35 cents per gross. The dealer charges 5 cents per dozen. How much does a manual training teacher save on a gross by buying by the gross than by the dozen?

5. Why must a dealer charge a higher rate in selling in small quantities than in large quantities?

6. How much does a grocer receive for a crate of eggs at 28 cents per dozen?

7. If bananas are selling at 30 cents per dozen, how many should I receive for 10 cents?

8. Lead pencils are quoted at $3.60 per gross. How much profit is made on a gross by selling them at 5 cents each?

9. A grocer paid his customers 26 cents per dozen for eggs. He sold them at $8.00 per case. What was his profit on each case of eggs, deducting 20 cents for the cost of the case and the fillers?

10. A school supply store buys pens at 75 cents per gross The retail price of these pens is 6 for 5 cents. Find the gain on a gross of these pens.

Exercise 6. Paper Measure

Writing paper is usually cut into sheets which are sold by the ream or by the pound. In counting paper the following table is used:

24 sheets = 1 quire[1]
20 quires = 1 ream

1. How many sheets of paper are there in a ream?

In order to provide extra sheets to allow for waste in printing, 500 sheets are now usually considered as a ream. How many sheets per hundred are allowed for waste?

2. How many sheets are there in $\frac{1}{2}$ ream?

3. If paper costs $1.25 a ream, how much is the cost of each sheet? At that rate how many sheets can be bought for a dime?

4. A certain grade of paper weighs 6 pounds per ream. How many sheets would one receive in buying 1 pound of this paper?

5. How many sheets of paper are there in a pound if a ream weighs 4 pounds?

6. Which is cheaper, 24 sheets of typewriter paper weighing 5 pounds per ream at the rate of 2 sheets for 1 cent, or $\frac{1}{4}$ of a pound of this paper for 10 cents?

7. How many sheets of paper are there in 5 quires; in $\frac{1}{2}$ quire; in 8 quires?

8. Why is paper now usually sold by the pound rather than by counting the sheets?

9. A certain grade of paper weighs 5 pounds to the ream. What is the weight of a quire of that paper? How much does each sheet weigh?

---

[1] The term quire is now used mainly in selling foolscap paper which is put up in folded sheets.

Measures of Time

The clock is the standard instrument for measuring time.

The earth turns round on its axis once in a day. For convenience in estimating shorter periods of time, a day is divided into hours, minutes and seconds. In measuring longer periods of time the days are grouped into weeks, months and years.

Table of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
28, 29, 30 or 31 days[1] = 1 month (mo.)
365 days = 1 year
366 days = 1 leap year

The earth travels around the sun in approximately 365¼ days. In order to save the inconvenience of having a fractional part of a day left each year, we provide an extra day every 4 years. We call a year containing this extra day a "leap year."

10 years are known as a decade.
100 years are called a century.

---

[1] The following rhyme will help you in remembering the number of days in each month:

"Thirty days hath September
April, June and November.
All the rest have thirty-one
Except the second month alone
Which has but twenty-eight in fine[2]
'Till leap year gives it twenty-nine."

[2] The old expression in fine meant in small amount.

Exercise 7

1. How many minutes are there in 4 hours?

2. How many seconds are there in forty minutes?

3. In a certain city school system the school day in the high school is divided into 7 forty-five minute periods. How many hours is the high school in session, not counting the noon intermission?

4. In Belgium the clocks mark ever hour from 1 to 24. Which is better, the Belgian plan or ours?

5. When a clock face is divided into twelve hour spaces, each one is how much larger than one of the spaces on a twenty-four hour clock face of the same size?

6. Find the number of seconds in a day.

7. What is meant by the expression 9:00 A. M.; by 3:30 P. M.?

8. Fifteen minutes each day devoted to some plan of self-improvement will amount to how much time in a year?

9. A train running 40 miles an hour goes how many feet per minute? How many feet per second?

10. The Mauretania made the voyage from Queenstown to New York, a distance of 2800 miles, in 4 days, 10 hours and 48 minutes. Find the total number of minutes required for this voyage across the Atlantic Ocean.

11. The United States census is taken once every decade. How many times is a census taken in a century?

12. Any year divisible by 4 is a leap year except the century years, which must be divisible by 400 to be a leap year. Which of the following will be a leap year: 1918, 1920, 1926, 1960, 2000?

Exercise 8. Reduction of Denominate Numbers

Numbers expressing measures are called *denominate numbers*. For example: 3 quarts, 4 feet, 8 cubic feet, 3 bushels, 7 acres and 15 pounds are denominate numbers.

1. A cow gives 3 gallons of milk at each milking. How many pint bottles can be filled from one milking?

In finding the number of pints in 3 gallons, we have changed or reduced *from higher units to lower units.* In this process we have *multiplied* the number of higher units (3) by the number of the lower units (8) in *one* of the higher units.

Reduce:

2. 4 bushels to pecks; to quarts; to pints.

3. 6 gallons to quarts; to pints; to half-pints.

4. 10 feet to inches; 5 feet to inches.

5. 2 rods to yards; to feet; to inches.

6. 2 tons to pounds; 8 pounds to ounces

7. 5 feet 7 inches to inches.

Solution: 5 feet=5×12 inches=60 inches. 60 inches+7 inches=67 inches.

8. 10 feet 6 inches to inches.

9. 2 yards 3 feet to feet.

10. 1 ton 500 pounds to pounds.

11. 2 bushels 3 pecks to pecks.

12. 5 gallons 2 quarts to pints.

13. 5 yards 6 inches to inches.

14. 2 pounds 9 ounces to ounces.

15. 4 bushels 3 quarts to quarts.

16. 2 miles 80 rods to rods.

17. 3 pecks 5 quarts to quarts.

18. 4 cubic feet to cubic inches.

19. 3 quarts 1 pint to pints.

20. 4 rods 3 yards to yards.

21. 2 square feet 24 square inches to square inches.

22. Find the number of inches in a mile.

23. A bin measured 9 feet 4 inches long; 6 feet 8 inches wide and 5 feet 11 inches deep. Express these measurements in inches. Find the number of cubic inches in the bin.

Exercise 9

1. A school room measured 24 feet in width and 30 feet in length. Express the width and length of this room in yards.

In finding the number of yards in 24 feet, we have changed or reduced *from lower units to higher units* of measure. In this process we *divided* the number of lower units (24) by the number of lower units (3) making a higher unit.

Reduce:

2. 156 inches to feet.

3. 48 ounces to pounds.

4. 56 quarts to pecks.

5. 54 square feet to square yards.

6. 10,368 cubic inches to cubic feet.

7. 48 pints to gallons.

8. 22 yards to rods.

9. 64 quarts to bushels.

10. 320 square rods to acres.

11. 54 cubic feet to cubic yards.

12. 195 inches to feet and inches.

Solution: 195 inches ÷ 12 inches = 16, with a remainder of 3 inches.
Then 195 inches = 16 feet 3 inches.

Change to higher units:

13. 54 inches

14. 87 ounces

15. 46 yards

16. 61 pints

17. 192 minutes

18. 1632 rods

19. 67 cubic feet

20. 137 square feet

21. 504 square inches

22. 27 pecks

23. 89 feet

24. 975 seconds

25. Washington monument is 555 feet high. Express its height in yards and feet.

26. A field measured 36 rods long and 24 rods wide. Find the area of this field in square rods. Express the area in acres and square rods.

27. A load of coal weighed 3725 pounds. Express the weight of this load in tons and pounds.

Exercise 10

1. A dairyman bought 15 gallons of milk from a neighbor. How many quart bottles will that amount of milk fill? How many pint bottles will it fill?

2. This dairyman has customers who use 186 quarts and 122 pints of milk per day. How many gallons does he need each day to supply these customers?

3. A 10-pound box of salted peanuts is put into 4-ounce bags. How many such bags can be filled from the box?

4. During one month a family used $10\frac{1}{2}$ gallons of milk. How much was their milk bill at 10 cents per quart?

5. A grocer sells a barrel of vinegar at 30 cents per gallon. How much does he receive for the vinegar?

6. If you need 6 yards of trimming for a dress and have bought 54 inches, how much more will you need to purchase?

7. Gerald picked a $2\frac{1}{2}$-gallon bucket full of wild blackberries and sold them at 10 cents per quart. How much did he receive for them?

8. If I drink a pint of milk every day, how many gallons shall I drink in a year?

9. Last spring a grocer sold 128 quarts of onion sets. How many bushels did he sell?

10. A farmer measured a field and found that it contained 6400 square rods. How many acres were there in the field?

11. How many crates of berries, 16 quarts to a crate, are used by a bakery in making 64 pies, allowing 1 pint of berries for each pie?

12. A farmer needs 10 bushels of seed corn to plant 40 acres of corn. How many quarts of seed corn are needed per acre?

13. A truck gardener plants 5 pounds of onion seed per acre. How many ounces of onion seed shall I need to plant a plot 4 rods long and 2 rods wide?

14. How much profit does a grocer make on a quart of onion sets if he buys them at $2.40 a bushel and sells them at $12\frac{1}{2}$ cents a quart?

In working with denominate numbers, we find a change from abstract numbers which do not represent measures. Instead of there being 10 inches in a foot, there are 12. Instead of 10 feet making a yard, 3 feet make a yard. That is, the number of units of a lower denomination that is required to make a unit of a higher denomination changes with the denomination that we are using.

Exercise 11. Addition of Denominate Numbers

1. A rectangular room is 13 feet 6 inches long and 10 feet 8 inches wide. Find the distance around the room.

Feet	Inches
13	6
13	6
10	8
10	8
48	4

There are two sides, each 13 feet 6 inches, and two ends, each 10 feet 8 inches. Adding the number of inches=28. 28 inches=2 feet 4 inches. Place the 4 under the inches column and add the 2 feet to the numbers in the feet column, making 48 feet. The distance around the room is 48 feet 4 inches.

Add the following:

2. 4 feet 3 inches; 5 feet 2 inches; 3 feet 9 inches and 7 feet 8 inches.

3. 2 pounds 5 ounces; 3 pounds 3 ounces; 2 pounds 11 ounces and 4 pounds 1 ounce.

4. 3 bushels 2 pecks; 2 bushels 3 pecks; 3 bushels 1 peck and 5 bushels 1 peck.

5. 1 square foot 42 square inches; 2 square feet 88 square inches and 4 square feet 112 square inches.

6. 3 quarts 1 pint; 2 quarts 1 pint; 6 quarts 1 pint and 5 quarts 1 pint.

7. 3 rods 3 yards; 4 rods 3 yards, 3 rods 2 yards and 5 rods $1\frac{1}{2}$ yards.

8. A rectangular picture frame is 1 foot 10 inches long and 1 foot 3 inches wide. How much picture molding did it take to make this frame, the corners being lapped?

9. A grocer sold one customer 2 bushels 3 pecks of seed potatoes; to another 1 bushel 2 pecks and to a third 3 pecks. How many seed potatoes did he sell to the three cutomers?

10. The three sides of a triangular flower bed measure 7 feet 8 inches; 6 feet 9 inches and 5 feet 11 inches. What is the distance around the flower bed?

Exercise 12. Subtraction of Denominate Numbers

1. From a bolt of cloth containing 18 yards 8 inches a clerk cut a piece 7 yards 18 inches. How much was left?

Yards	Inches
18	8
7	18
10	26

We can not subtract 18 inches from 8 inches, so we must change one of the 18 yards to inches, making 36 inches. 36 inches + 8 inches = 44 inches. 44 inches — 18 inches = 26 inches. 17 yards — 7 yards = 10 yards. 10 yards 26 inches may be expressed as 10 yards 2 feet 2 inches.

Subtract the following:

2. 10 bushels 3 pecks − 8 bushels 2 pecks.

3. 5 gallons 2 quarts − 4 gallons 3 quarts.

4. 11 feet 6 inches − 5 feet 4 inches.

5. 39 tons 500 pounds − 15 tons 1750 pounds.

6. 4 rods 12 feet − 2 rods 15 feet.

7. 8 pounds 8 ounces − 6 pounds 4 ounces.

8. 10 square feet 124 square inches − 2 square feet 24 square inches.

9. 16 acres 80 square rods − 10 acres 120 square rods.

10. 15 bushels 5 quarts − 10 bushels 17 quarts.

11. A room is 15 feet 8 inches long and 11 feet 10 inches wide. How much does its length exceed its width?

12. The world's record for the running broad jump is 24 feet 11$\frac{3}{4}$ inches and for the standing broad jump 11 feet 6 inches. How much greater was the leap in the running broad jump than in the standing broad jump?

13. A grocer bought a load of potatoes which measured 32 bushels 2 pecks. He sold 42 pecks to customers during the first day. How many potatoes did he have left out of the load?

Exercise 13. Multiplication of Denominate Numbers

1. A gardener wishes to put a board border around a triangular flower pot, each side being 4 feet 7 inches in length. How many feet long must he buy a board to go around the plot, not allowing for waste in sawing?

4 feet 7 inches
3
13 feet 9 inches

3×7 inches=21 inches. 21 inches=1 foot 9 inches. 3×4 feet=12 feet. 12 feet+1 foot=13 feet. He would need a board 13 feet 9 inches long.

Multiply the following:

2.
1 bushel 3 pecks
5

3.
3 yards 2 feet
4

4.
1 ton 1500 pounds
2

5.
10 feet 8 inches
16

6.
12 square yards 4 square feet
3

7.
2 pounds 12 ounces
7

8.
3 quarts 1 pint.
6

9.
3 yards 6 inches
18

10.
4 gallons 3 quarts
5

11. A square room measures 11 feet 6 inches on each side. How much picture molding would it take to go around the room, counting out space for two windows each 4 ft. wide?

12. A cubic foot of water weighs 62 pounds 8 ounces. What is the weight of the water in a tank 4 feet×3 feet×2 feet?

13. The length of each side of a rug in the shape of a hexagon (6-sided figure) is 1 foot 7 inches. How much binding tape would it take to go around the edge of this rug?

14. In running a touring car a man used on an average of 8 gallons 3 quarts of gasoline per day. How much did he use in a week?

15. A wholesale firm packs 24 boxes of a certain brand of rolled oats into a shipping crate. If each box contains 1 pound 12 ounces of rolled oats, how many pounds of oats are used for one crate?

Exercise 14. Division of Denominate Numbers

1. Two girls bought a remnant of gingham 3 yards 8 inches long. How long was each piece if they divided it equally?

$$\begin{array}{r|l} & \text{1 yard 22 inches} \\ \hline 2 & \text{3 yards 8 inches} \end{array}$$

3 yards ÷ 2 = 1 yard with a remainder of 1 yard. 1 yard = 36 inches. 36 inches + 8 inches = 44 inches. 44 inches ÷ 2 = 22 inches. Each piece was 1 yard 22 inches or 1 yard 1 foot 10 inches long.

Divide:

2. 17 yards 2 feet by 2.

3. 4 pounds 8 ounces by 3.

4. 25 yards 2 feet 9 inches by 3.

5. 5 hours 30 minutes by 2.

6. 1 pound 8 ounces by 4.

7. 12 rods 4 yards by 5.

8. 20 pounds 6 ounces by 6.

9. 2 yards 2 feet by 4.

10. 5 cubic feet 172 cubic inches by 4.

11. 4 yards 2 feet 8 inches by 2.

12. 5 hours 20 minutes by 8.

13. 10 feet 3 inches by 3.

14. 13 feet 8 inches by 4.

15. 6 pounds 4 ounces by 10.

16. The distance around a square bin is 22 feet 8 inches. What is the length of each of the 4 sides?

17. The distance around an equilateral (equal-sided) triangle is 10 feet 3 inches. What is the length of each of the three equal sides?

18. Louise cut a strip of toweling 3 yards 1 foot long into 3 equal lengths for towels. How long was each towel before it was hemmed?

19. A table measures 1 foot 8 inches each way. I wish to make a checker board of the top. There are 8 squares each way. How long can I make each square?

20. A poultry raiser sold 6 spring chickens weighing 13 pounds 14 ounces. What was the average weight of each chicken?

21. A board 11 feet 8 inches long is sawed into 2 equal lengths. How long is each part, not allowing for waste?

22. A truck farmer puts 8 bushels 1 peck in 3 barrels. What was the average amount put into each barrel?

23. A pail of butter containing 5 pounds 4 ounces was divided equally by two women. How much butter did each receive?

24. The sum of the four sides of a square is 18 feet 4 inches. Find the length of one side.

25. The sum of the three equal sides of a triangle is 16 feet 3 inches. Find the length of one side.

26. Show how problem 25 can be solved by first reducing to inches, then dividing by 3 and reducing the quotient to feet and inches.

Exercise 15. Cotton

Applied Problems in Weights and Measures

Spraying growing cotton to destroy injurious insects

1. About 3 pecks of cotton seed are required to plant an acre. How many bushels of cotton seed are used in planting 20 acres of cotton?

2. Cotton is drilled in rows and the plants are then thinned out so that they are 12 to 24 inches apart according to the kind of land. How many hills are there in a row a quarter of a mile long when thinned 18 inches apart?

3. There are 11 rows a quarter of a mile long in an acre of cotton when the rows are 3 feet apart. How many hills are there in an acre when thinned 18 inches apart?

4. A laborer picks on the average about 375 pounds of cotton per day. What are his daily wages at 80 cents per hundred pounds? How long will it take him to pick a ton of cotton?

5. Black prairie land yields about 800 pounds of cotton per acre. How many pounds of cotton are obtained from a square rod at that rate?

6. How many tons of cotton can be raised on 20 acres at the rate of 800 pounds per acre? How many 500 pound bales?

7. At 70 cents per hundredweight, how much does it cost to gin the cotton raised on 20 acres yielding 800 pounds an acre?

8. What is the value of the cotton produced on an acre if the yield is 800 pounds and the lint cotton sells for 11 cents per pound?

Exercise 16. Oil

Applied Problems in Weights and Measures

An Oklahoma "Gusher"

1. In 1915 the production of petroleum or crude oil totaled 281,104,104 barrels. A barrel of petroleum is equal to 42 gallons. How many gallons were produced that year?

2. In a certain oil region the wells vary in depth from 600 feet to 3300 feet. Express the depth of these wells in yards.

3. It costs about $5.00 per foot[1] to drill an oil well and 50 cents per foot, extra, for the tubing. Find the cost of drilling and tubing a well 1500 feet deep.

4. The number of oil wells per acre varies according to the amount of oil under the territory. At the rate of 2 wells per acre, how many wells could be sunk in a farm 80 rods long and 60 rods wide?

5. The landowner generally receives $\frac{1}{8}$ of the production of the oil as a royalty for his land. How much would his royalty amount to in a year on 5 wells, each flowing at the rate of 1000 barrels per day?

6. A well flowing 1000 barrels per day will, in one day, fill how many car tanks holding 8000 gallons each?

7. Crude petroleum is usually worth about $1.00 per barrel of 42 gallons. Find the cost per gallon of crude petroleum.

8. If the life of an oil well is 2 years and it has an average flow of 3000 gallons per day for that time, how much is the value of the product of such a well at $1.00 per barrel for the crude oil?

---

[1] This cost varies with the depth and structure of the earth at the point where the well is drilled.

Matches

Applied Review Problems

A Dipping Machine.[1] Photo. Brown Bros., N.Y.

Matches are less than a century old. Now they are one of the necessaries of life. The quantities used, the amount of timber required and the *efficiency* of the machines employed in their manufacture are interesting and instructive problems.

1. Counting each match $\frac{1}{8}$ inch square and $2\frac{2}{5}$ inches long, how many matches would a cubic foot of white pine timber produce, not allowing for waste?

2. Mr. Franklin has a pine tree on his farm from which $1\frac{1}{4}$ cords of match timber can be cut. Allowing $\frac{1}{5}$ for waste, what number of matches would this tree produce?

3. How many matches will $3\frac{1}{2}$ cords of pine timber produce, allowing $\frac{1}{5}$ for waste?

4. Mr. Warren has a poplar tree which he estimates will produce $2\frac{1}{2}$ cords of match timber. What number of matches would this equal, counting $\frac{1}{5}$ for waste?

5. How many matches can be produced from $5\frac{3}{4}$ cords of poplar timber, counting $\frac{1}{5}$ for waste?

---

[1] "Officially known as the fastest machine in the world. It takes forty minutes to turn out a million matches."—From the "How and Why" Library.

6. Open the ends and sides of an empty paper match box so that it may be laid out flat, and find the number of square inches of strawboard used in its construction.

7. Find the weight of an empty match box (strawboard), and estimate the number of such boxes that can be produced from a ton of strawboard.

8. If it takes 10 tons of straw to make 7 tons of finished strawboard, how much straw will be required to make 1 ton of strawboard?

9. If baled straw, delivered at a factory, is worth $7.50 per ton, what would the straw in one ton of strawboard cost?

10. A certain machine can make 500 match boxes per minute.[1] How many match boxes can this machine turn out in an 8-hour day?

11. How many boxes is this per second?

12. How many square feet of strawboard does it require for this number of boxes?

13. Each of these boxes will hold 160 matches. What is the capacity, in matches, of one of these box making machines in an 8-hour day?

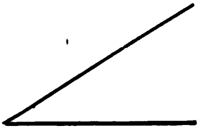
14. It is estimated that the "dipping" machine (to put heads on) in a certain factory will hold 1,000,000 matches at one time. If each load is completed in 40 minutes, how many finished matches will this machine produce in an 8-hour day?

---

[1] "An endless roll of brown strawboard is fed in a broad sheet to a machine that cuts it into strips wide enough to make the four sides of a sliding box cover, or the bottom and sides of the box. Through one machine after another these strips go. The box cover is given four folds and pasted into a square-sided endless tube. The tube goes through a special printing press that prints the top and bottom and one side, and pastes a strip of sand paper on the fourth side. Then the printed tube is cut into box lengths."—From the "How and Why" Library.

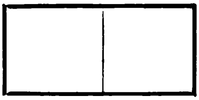
CHAPTER III

PRACTICAL MEASUREMENTS

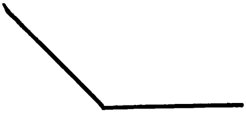


An angle is a figure formed by two lines meeting at a point.

Take a sheet of paper and fold one portion of an edge over on the other portion. Then crease across the paper and open the sheet of paper. The crease will form two angles with the edge of the paper. How do these angles compare in size? When one line meets another line so that the two angles formed are equal, the angles are called ***right angles.***



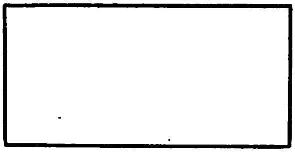
An angle that is smaller than a right angle is called an *acute* angle. The angle at the top of the page is an acute angle. Draw an acute angle on the blackboard.



An angle that is larger than a right angle is called an *obtuse angle.* The angle at the left is called an obtuse angle. Draw an obtuse angle on the blackboard.

These different kinds of angles will be useful in describing rectangles, parallelograms, triangles and many other figures.

Exercise 1. Rectangles



1. How many straight lines form the rectangle shown at the left?

2. How many angles has a rectangle?

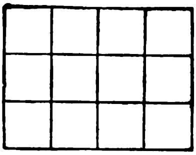
3. What kind of angles are these four angles?

A rectangle is a figure bounded by four straight lines and having four right angles.

4. Name various things in the room which are rectangular in shape. For example: the blackboard, the window panes.

5. What kind of units of measure do we use in describing the area or size of a rectangle?

Area of a Rectangle



6. What is the area of a rectangle 4 feet long and 3 feet wide?

7. How many square feet are there in one row a foot wide along the base?

8. How many such rows are there in the rectangle? Then how many square feet are there in the area of the rectangle?

In the same way show how to find the area of a rectangle:

9. 12 in. long and 8 in. wide.

10. 15 ft. long and 12 ft. wide.

11. 8 yd. long and 5 yd. wide.

12. 40 rods long and 20 rods wide.

13. In each of these rectangles how does the *number of squares in one row* compare with the number units of length in the base?

14. How does the *number of rows of squares* compare with the number of units of length in the width?

We have already seen that the area of a rectangle is found by multiplying the number of squares in one row by the number of rows. This is often expressed in the shorter form:

The area of a rectangle is equal to the product of its base and altitude.[1]

---

[1] The line representing the length of the rectangle is usually called the **base** and the line representing the width is called the **altitude.**

Exercise 2

1. The length of a blackboard is 12 feet and its width is 4 feet. What is the area of the blackboard in square feet? What is its area in square yards?

2. A slate blackboard, 3½ feet wide, extends across one end of a schoolroom and half way along the side. The room is 28 feet long and 24 feet wide. Find the number of square feet of slate in the blackboard. How much will this blackboard cost at 15 cents per square foot?

3. A blueberry patch 20 rods long and 12 rods wide yielded a profit of $450. Find the number of acres in the area of the berry patch and the profit per acre.

4. A city lot 150 feet long and 50 feet wide cost $2500. Find the number of square feet in the area of the lot and the cost per square foot.

5. The floor of a bathroom 8 feet 4 inches long and 6 feet 2 inches wide is covered with tiles an inch square. How many tiles were required to lay the floor?

6. An acre must contain 160 square rods. State the different dimensions which an acre may have. For example, it may be 16 rods long and 10 rods wide.

A square is a rectangle with equal sides.

7. A field is 40 rods square. How many acres are there in the field? If the yield of oats on this field was 650 bushels, what was the yield per acre?

It should be explained to the pupils that the principle on page 214 is used as a matter of custom and of convenience. Of course, there is really no such thing as "the product of its base and altitude," since a multiplier must always be an abstract number. What is meant is that the number expressing the square units of a rectangle is equal to the number of units of its length multiplied by the number of units of its breadth. It is the abstract numbers that are multiplied together, and not the length and the breadth. This will apply to other similar statements.

8. A kitchen is 9 feet long and 8 feet wide. How much will the linoleum cost to cover this floor at $1.30 per square yard?

9. The area of a rectangle is 208 square inches. It is 16 inches long. How wide is it?

Suggestion: How many inch squares are there in one row along the length? Since the whole rectangle contains 208 inch squares, how many rows of squares are there?

10. The sidewalk in front of a city residence is 4 feet wide. If the lot has a frontage of 45 feet, how much did the walk cost at $1.50 per square yard?

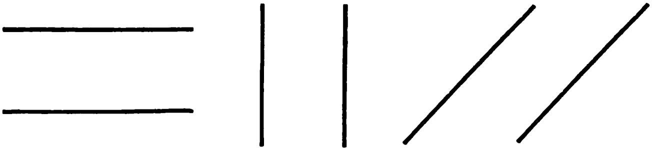
11. A tennis court is 78 feet long and 36 feet wide. How many square feet of space are used in laying out a tennis court? An acre contains 43,560 square feet. What decimal part of an acre is the area of the tennis court?

12. The area of a rectangle is 48 square feet. The length is 8 feet. What is the width?

13. A yield of 899 bushels, of corn was obtained from a field 60 rods long and 36 rods wide. What was the average yield per acre from this field?

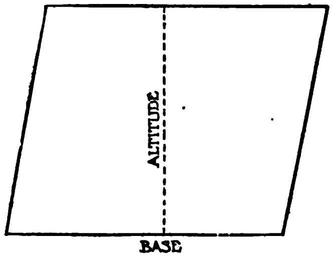
Exercise 3. Parallelograms

Parallel Lines



Two straight lines are *parallel* if they are the same distance apart throughout their length. The above figures show three pairs of parallel lines drawn in different positions.

Parallel lines will not meet, however far they may be extended.



1. How are the opposite sides of this parallelogram drawn?

> **A parallelogram is a four-sided figure with two pairs of parallel sides.**

Any side of a parallelogram may be considered as the *base.* The lower side is indicated as the base of the parallelogram in the figure shown above.

2. What kind of angles does the dotted line make with the base? This dotted line is called the *altitude* and represents the distance between the base and the opposite side.

Area of a Parallelogram

Cut a parallelogram out of a piece of tablet paper. Fold part of the base over on itself and crease a line for the altitude. Cut or tear the parallelogram along this altitude into two parts.

Show how these two parts can be arranged to form a rectangle. How does the base of the rectangle compare with the base of the parallelogram? Note that the creased line is the altitude of both the parallelogram and the rectangle.

The area of the rectangle is equal to the product of its base and altitude. The area, the base and the altitude of the parallelogram are each equal to the area, base and altitude of the rectangle. Thus we can see that:

> **The area of a parallelogram is equal to the product of its base and altitude.**

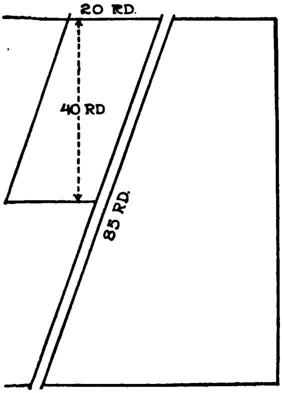
A rectangle is also a parallelogram. The figure shown above is sometimes called a rhomboid to distinguish it from a rectangle or oblong. A rhomboid with equal sides is also called a rhombus.

Exercise 4

1. Find the area of a parallelogram with a base of 11 inches and an altitude of $8\frac{1}{2}$ inches.

2. What is the area of a parallelogram with a base of $1\frac{2}{3}$ feet and an altitude of $1\frac{1}{3}$ feet?

3. A truck garden in the shape of a parallelogram has a base of 24 rods and an altitude of 15 rods. How many acres are there in the garden?



4. A diagonal road, 2 rods wide, runs across a man's farm. The road measures 85 rods in length. How many acres does this road take out of the farm?

Suggestion: The length may be taken as the base of the parallelogram thus formed and the altitude as the width of the road.

5. How many acres are there in the field at the left of the road with the dimensions as shown in the illustration?

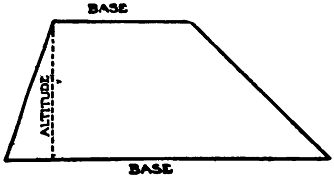
6. A flower bed in the shape of a parallelogram has a base of 3 yards and an altitude of 7 feet. Find the area of the flower bed in square feet.

7. A board in the shape of a parallelogram is 6 feet long and 8 inches wide. How many square feet are there in the area of one side?

8. A field in the shape of a parallelogram is 30 rods long and 16 rods wide. How much is the field worth at $150 an acre?

9. A field in the shape of a parallelogram is $16\frac{1}{2}$ rods long and $12\frac{3}{4}$ rods wide. Find its area in acres.

Exercise 5. Trapezoids

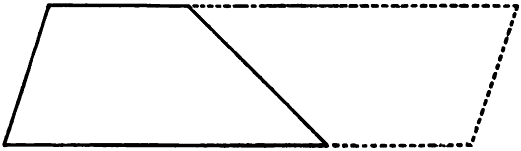


The trapezoid at the left has how many pairs of parallel sides?

A trapezoid is a four-sided figure with only one pair of parallel sides.

The two parallel sides are called the *bases* of the trapezoid, and the distance between them the *altitude*. The altitude must make what kind of angles with the base?

Area of a Trapezoid



Draw a trapezoid on paper and cut it out. Make a second trapezoid exactly the same size as the first, using the first as a pattern.

Arrange the second trapezoid in the position shown by the dotted lines. What shaped figure is formed?

1. How long is the base of this parallelogram?

The altitude of the parallelogram is the same as the altitude of the trapezoid.

The area of the parallelogram is equal to the sum of the two bases of the trapezoid multiplied by the altitude.

The trapezoid is what part of the area of the parallelogram?

Show then that the following statement is true:

The area of a trapezoid is equal to $\frac{1}{2}$ the sum[1] of the two bases multiplied by the altitude.

---

[1] One-half the sum of the two bases may be called the **average base.**

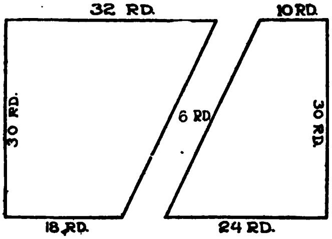
2. Find the area of a trapezoid with two bases equal to 7 inches and 15 inches and an altitude of 6 inches.

Solution: 7 inches + 15 inches=22 inches. $\frac{1}{2}$ of 22 inches=11 inches, the average base. 11×6=66. Therefore the area of the trapezoid=66 square inches.

3. What is the area of a trapezoid with bases of 40 rods and 20 rods and an altitude of 30 rods?

Find the areas of the trapezoids having the following dimensions:

Upper Base	Lower Base	Altitude
4. 15 inches.	21 inches.	16 inches.
5. 40 rods.	45 rods.	32 rods.
6. 10 feet.	16 feet.	11 feet.
7. $1\frac{2}{3}$ feet.	$2\frac{1}{4}$ feet.	$\frac{7}{8}$ feet.
8. $9\frac{1}{2}$ inches.	$13\frac{3}{4}$ inches.	$9\frac{7}{8}$ inches.
9. $32\frac{1}{2}$ rods.	$48\frac{3}{4}$ rods.	40 rods.
10. $\frac{7}{8}$ feet.	$1\frac{1}{3}$ feet.	$\frac{3}{4}$ feet.
11. 8 rods.	42 rods.	40 rods.

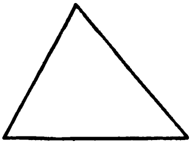


12. A railroad cuts a field into two trapezoids with dimensions as shown in the diagram. Find the area of each field in square rods. Find the number of acres in each field.

13. A board 12 feet long is 8 inches wide at one end and 12 inches wide at the other. Find the number of square inches in the area of the board.

14. Bring to the class problems which are based upon the measurements of fields or other objects in your community that have the shape of a trapezoid.

Exercise 6. Triangles

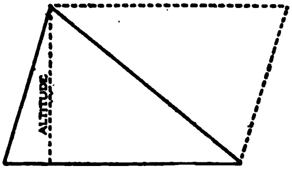


How many sides has a triangle? How many angles? *Tri*-angle means *three* angles.

A triangle is a figure bounded by three straight lines or sides.

Give examples of triangles which you have noticed in any of the objects in your community, such as the gable on a house.

Area of a Triangle



Draw a triangle of any shape. Then draw dotted lines parallel to two of the sides as shown in the illustration. What shaped figure is formed by the addition of the dotted lines?

Cut the parallelogram along the middle line and compare the sizes of the two triangles. The first triangle is what part of the area of the parallelogram?

The area of the parallelogram has been shown to be equal to the product of the base and the altitude.

The area of a triangle is equal to one-half the product of its base and altitude.[1]

Exercise 7

1. Find the area of a triangle with a base of 8 inches and an altitude of 7 inches.

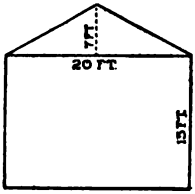
Solution: The product of the base and altitude $=56$. $\frac{1}{2}$ of the product of the base and altitude $=\frac{1}{2}\times 56=28$. Therefore the area of the triangle $=28$ square inches.

---

[1] Some pupil may also be able to show that the area of a triangle is equal to one-half of the base multiplied by the altitude or one-half of the altitude multiplied by the base.

Find the areas of triangles having the following dimensions:

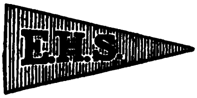
	Base	Altitude
2.	12 inches.	9 inches.
3.	7 feet.	2 yards.
4.	15 feet.	11 feet.
5.	20 rods.	12 rods.
6.	$\frac{3}{4}$ feet.	$\frac{2}{3}$ feet.
7.	$1\frac{2}{3}$ feet.	$1\frac{1}{4}$ feet.



8. The width of the gable of a barn is 20 feet and the altitude is 7 feet. Find the number of square feet in the area of the gable. Find the total area of the end of the barn, using the dimensions shown in the illustration.

9. A triangular plot of ground has a base of 20 rods and an altitude of 16 rods. How many acres are there in this plot of ground?

10. A triangular field is 20 rods long and its greatest width is 8 rods. How many square rods does it contain? Express the area of this field in acres.



11. A pennant in the shape of a triangle is 24 inches long and 8 inches wide at one end. How many square inches of felt are there in the pennant?

12. The gable of a house is 28 feet wide and 14 feet high. How many square feet are there in the area of this gable? Draw a diagram to show the shape of the gable.

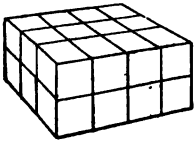
13. A triangular city lot is 166 feet long and 90 feet wide. How many square feet does it contain? What part of an acre (43,560 sq. ft.) is it?

14. The width of the gable of a barn is 30 feet and the altitude 12 feet. Find the number of square feet in the gable of this barn.

Exercise 8. Volumes of Rectangular Solids

A solid is a figure having three dimensions, length, breadth and depth (or thickness).

A *rectangular solid* is a solid with six rectangular faces.



1. Find the volume of a rectangular solid 4 feet long, 3 feet wide and 2 feet high.

2. How many square feet are there in the base of this solid?

3. How many cubic feet are there in a layer one foot deep?

4. How many such layers are there in the solid?

5. How many cubic feet are there in the volume of the rectangular solid?

The volume of a rectangular solid is equal to the number of cubic units in a layer one unit high, multiplied by the number of layers.

This principle is often expressed in the shorter form:

> **The volume of a rectangular solid is equal to the product of the three dimensions.**

6. Find the number of cubic feet in a room 12 feet long, 9 feet wide and 9 feet high. 12 cubic feet of air weigh a pound. How much will the air in such a room weigh?

7. An iceman brought me a rectangular piece of ice 11 inches long, 11 inches wide and 10 inches high for 50 pounds. Find the fraction of a cubic foot in this piece of ice. If ice weighs 57.5 pounds per cubic foot, how much under weight was the piece?

8. A farmer measured a rectangular crib of corn on the ear. The length was 16 feet 4 inches, the width 10 feet 7 inches and the depth of the corn 8 feet 2 inches. Find the number of bushels of corn in this crib, counting 4000 cu. in. per bushel.

9. Find the number of bushels of wheat (2150.42 cubic inches) in a bin 11 feet 5 inches long, 8 feet 6 inches wide and 7 feet 8 inches deep?

10. Find the number of bushels of wheat which can be loaded into a freight car 37 feet 6 inches long, 8 feet 6 inches wide and 5 feet in depth?

11. How many tons of coal can be loaded into a flat car 40 feet long, 8 feet 6 inches wide and 4 feet 6 inches in depth, allowing 38 cubic feet per ton?

12. Find the cost of excavating a basement 22 feet 6 inches long, 18 feet wide and 4 feet 6 inches deep at 50 cents per cubic yard.

13. Find the number of gallons (231 cubic inches) in a rectangular tank 6 feet long, 2 feet 2 inches wide and 2 feet 4 inches deep.

14. A hay mow 30 feet long and 16 feet 6 inches wide is filled with hay to an average depth of 10 feet. How many tons of hay are there in this mow, allowing 512 cubic feet per ton?

Exercise 9. Bin Problems

1. Find the number of bushels of wheat in a bin 15 feet × $6\frac{1}{4}$ feet, filled to a depth of $5\frac{1}{2}$ feet (2150.42 cubic inches per bushel or $1\frac{1}{4}$ cubic feet per bushel.)

2. How many tons of hard coal can be put in a bin $10\frac{1}{2}$ feet long, $5\frac{1}{2}$ feet wide and 4 feet deep, allowing 35 cubic feet of space per ton?

3. How many bushels of potatoes will a freight car hold that is $37\frac{1}{2}$ feet long, $8\frac{1}{2}$ feet wide, if filled to a depth of 6 feet?

A bushel of potatoes must be heaped to give full weight. A heaped bushel is equal to 2747.07 cubic inches.

4. A farmer built a potato bin 5 feet 6 inches long, 3 feet wide and 2 feet deep. How many bushels of potatoes did it hold when filled level full?

5. How many bushels of shelled corn can be put into a bin 6 feet 9 inches long, 3 feet 6 inches wide and 4 feet deep? (2150.42 cubic inches per bushel.)

6. Harry's father built a coal bin 12½ feet long, 7 feet wide and 5 feet high. At 35 cubic feet to the ton (for anthracite or hard coal), how many tons did this bin hold when filled?

7. Harry's uncle built a coal bin 14 feet × 8½ feet × 5 feet. How much anthracite coal did it hold?

8. Mr. Field bought 80 tons of bituminous (soft) coal for use at his factory. His coal room was 28 feet long and 17 feet wide. What was the average depth of the coal in this room, allowing 42½ cubic feet per ton?

9. Plan a bin to hold 10 tons of anthracite coal. Give length, width and depth. Prove that your bin will hold that amount.

10. Plan a bin to hold 20 bushels of potatoes. See Problem 3 for note on number of cubic inches in a bushel.

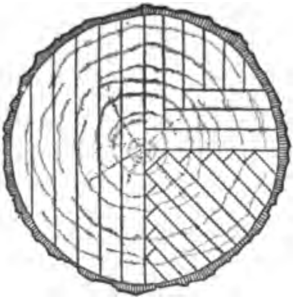
11. A bushel of corn in the ear occupies 4000 cubic inches of space. Find the number of bushels in a corn crib 16 feet 5 inches long, 8 feet 3 inches wide and filled to a depth of 10 feet 6 inches.

12. How many bushels of shelled corn can be put into a bin 10 feet 2 inches long, 7 feet 3 inches wide and 6 feet 10 inches deep?

13. Measure some bin and make a problem for class use based upon the actual measurements.

14. Plan a bin to hold 50 bushels of wheat.

Lumber Measure



Formerly sawlogs were sawed into boards without being turned. The left side of the illustration shows this method of sawing lumber. Valuable woods are now largely quarter sawed, the log being divided lengthwise into four quarters and the boards being sawed alternately from the sides of each quarter as shown in the upper right-hand quarter, or diagonally as shown in the lower right-hand quarter.

This method of quarter sawing is partly for the purpose of preventing the warping of the wood and partly to distribute the graining in the wood more evenly across the boards.

Show by the above diagram how the graining (made by the rings of growth) is better distributed in the sections that are quarter sawed.

Lumber is measured by what is known as the *board foot.* A board foot[1] is a piece of lumber one foot square and one inch thick. Any lumber less than an inch thick is usually considered as an inch thick in computing the number of board feet.

A board 6 feet long, 1 foot wide and 1 inch thick contains 6 board feet. A board 6 feet long, 6 inches wide and 1 inch thick contains 3 board feet. A piece of timber 6 feet long, 1 foot wide and 4 inches thick contains 24 board feet.

The number of board feet in a piece of lumber is equal to the number of square feet in one of its surfaces multiplied by the number of inches in thickness.

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[1] The teacher should be provided with a board 1 toot square and 1 inch thick to give the pupils a concrete conception of a board foot.

Exercise 10

1. Find the number of board feet in a board 14 feet long, 6 inches broad and 1 inch thick.

2. How many board feet are there in a plank 10 feet long, $1\frac{1}{4}$ feet broad and 2 inches thick.

3. A sill 16 feet long, 6 inches wide and 6 inches thick contains how many board feet?

4. How many board feet are there in 20 planks 12 feet long, 1 foot wide and $\frac{3}{4}$ inch thick?

Lumber in large amounts is reckoned in thousands of board feet. "3000 feet" of lumber or "3 M ft.," as it is usually written, means three thousand *board feet.*

5. What is the cost of 5 M ft. of hardwood flooring at $80 per M ft.?

6. Find the cost of 5 M ft. of barn siding at $40 per M ft.

In stating the dimensions of pieces of lumber it is customary to use the sign ′ to indicate *feet* and the sign ″ to indicate *inches.*

7. How many board feet are there in a sill 10′×8″×8″?

Find the number of board feet in the following:

8. 40 boards 12′×12″×2″.

9. 50 boards 16′×6″×1″.

10. 25 boards 16′×10″×2″.

11. 20 boards 16′×4″×$\frac{3}{4}$″.

12. 20 studding 12′×4″×2″.

13. 60 joists 20′×8″×2″.

14. 40 sills 20′×12″×10″.

15. 25 posts 12′×12″×10″.

16. 60 sleepers 20′×10″×3″.

17. 25 planks 12′×1′×2″.

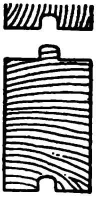
Find the cost of the following lumber:

18. 25 boards 16′×8″×1″ at $30 per M.

19. 16 boards 12′×15″×$1\frac{1}{2}$″ at $48 per M.

20. 35 boards 15′×6″×$\frac{3}{4}$″ at $32.50 per M.

Exercise 11. Carpentering Problems



When boards have tongues and grooves cut in them for flooring, part of the width of the board is wasted. In addition there is also some waste in squaring up the ends and sawing the boards to fit the space where they are used. For these reasons carpenters usually allow $\frac{1}{4}$ of the amount of lumber ordered for waste.

1. What will it cost for a maple floor 16′×30′ at $50 per M, allowing $\frac{1}{4}$ extra for waste?

Solution: 16×30×1 (the thickness)=480, the number of actual board feet needed. But $\frac{1}{4}$ extra must be allowed for waste in grooving the boards and laying the floor. $\frac{1}{4}$ of 480=120. 480+120=600. 600 board feet=.600 M ft. .6 M ft. at $50 per M=.6×$50=$30, the cost of the flooring.

2. What will it cost for a floor 30′×25′ at $55 per M, allowing $\frac{1}{4}$ for waste?

3. Find the cost of laying a plank walk 3 feet wide, 90 feet long and $1\frac{1}{2}$ inches thick at $28 per M.

4. A floor of a veranda 8′×20′ is laid of stock $1\frac{1}{2}$″ thick. How many feet of flooring are required, allowing 5 feet extra for squaring the ends?

5. The Homer Club erected a backstop for their baseball diamond $10\frac{1}{2}$′×24′. How many board feet of lumber 1″ thick was needed for the siding?

6. It also required 6 pieces of 2″×4″, $10\frac{1}{2}$′ long, for uprights and 6 pieces of 2″×4″, $7\frac{1}{2}$′ long, for braces. How many feet of 2″×4″ were needed?

7. How much did the lumber for the backstop cost at $30 per M?

8. Find the cost of flooring a barn 30′×24′ with 2″ planks at $30 per M.

Exercise 12. Shop Problems

Use the following price list in solving these problems:

Pine 6 cents. Gumwood 9 cents. Quartered oak 15 cents.

1. Find the cost of the following taboret made of quartered oak: Top, $16''\times 16''\times 1''$; bottom, $12''\times 12''\times 1''$; four legs, $24''\times 3''\times 1''$.

2. Find the cost of this footstool: Top, pine, $14''\times 7\frac{1}{2}''\times 1''$; sides, gum, $13\frac{1}{2}''\times 2\frac{1}{2}''\times 9''$; 4 legs, gum, $1\frac{1}{2}''\times 1\frac{1}{2}''\times 6''$. Imitation leather 10 cents; tacks 5 cents; screws 5 cents.

3. How much will this frame for a sign, made of pine, cost: 2 pieces $8'\times 2''\times 1''$; 2 pieces $1\frac{1}{3}'\times 2''\times 1''$?

4. Compute the cost of the gum lumber in the following candlestick: 1 piece $2''\times 2''\times \frac{3}{8}''$; 1 piece $1\frac{1}{2}''\times 1\frac{1}{2}''\times 2''$; 1 piece $4\frac{1}{2}''\times 4\frac{1}{2}''\times 1''$; 1 piece $3''\times 2''\times \frac{3}{8}''$.

5. Find the cost of quartered oak for a pedestal: 2 pieces $13''\times 3''\times 1''$; 4 pieces $2\frac{1}{2}''\times 2\frac{1}{2}''\times 1''$; 2 pieces $8\frac{1}{2}''\times 8\frac{1}{2}''\times 1''$; 2 pieces $32''\times 3\frac{1}{2}''\times \frac{3}{4}''$.

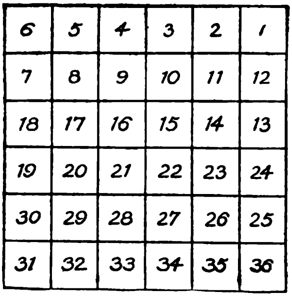
6. Find the cost of quartered oak for this cabinet for a wireless receiving set: 1 piece $18\frac{1}{2}''\times 11''\times 2''$; 2 pieces $10''\times 3\frac{1}{2}''\times 2''$; 1 piece $17\frac{1}{2}''\times 3\frac{1}{2}''\times 2''$.

7. How much will this lumber cost for a music cabinet: quartered oak, 3 boards $36''\times 14''\times \frac{3}{4}''$; 1 board $16''\times 17''$; 2 boards $3''\times 13\frac{1}{2}''$; 6 boards $1\frac{1}{2}''\times 13\frac{1}{2}''$; pine, 2 boards $13\frac{1}{2}''\times 9\frac{1}{2}''$; 10 boards $12\frac{1}{2}''\times 15\frac{1}{2}''$.

8. Find the cost of the following wood: 2 boards $14''\times 2\frac{3}{4}''\times \frac{1}{2}''$ at 12 cents a board foot; 2 boards $6''\times 6''\times \frac{1}{2}''$ at 12 cents a board foot; 1 board $14''\times 7\frac{1}{2}''\times 2\frac{1}{2}''$ at 6 cents a board foot.

9. Find the cost of the following taboret made of pine: 2 pieces $16''\times 16''\times \frac{3}{4}''$; 2 pieces $4''\times 16''\times \frac{3}{4}''$; 2 pieces $3\frac{1}{2}''\times 19''\times \frac{3}{4}''$.

Exercise 13. Measuring Land



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A Congressional Township

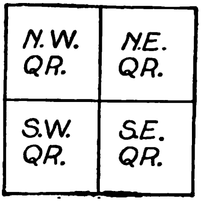
In most parts of our country the land is laid off in townships. These townships are 6 miles square.

1. How many square miles are there in a township?

Each square mile in a township is called a *section*. The sections of each township are numbered in the manner shown in the figure.

2. Draw a plan of a township showing the 36 sections and number them in the correct way without referring to the diagram in the book.

Each section or square mile is divided into quarter-sections for convenience in describing small farms. The quarter-sections are not numbered, but are called the Northeast, Northwest, Southeast and Southwest quarters.



N.W. QR.	N.E. QR.
S.W. QR.	S.E. QR.

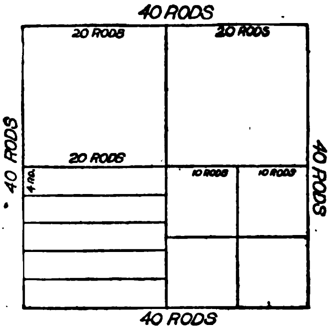
A Section, or Square Mile

3. How many acres are there in a section? How many in a quarter-section?

4. An 80-acre farm is what part of a quarter-section? Of a section?

5. A 40-acre farm is what part of a quarter-section? Of a section?

6. Roads are generally built to follow section lines. How far is it by section roads from the lower right-hand corner of section 26 to the upper left-hand corner of section 16? Show at least 3 ways in which one could go by road from the first to the second point.



A truck farmer owns a tract of land with the dimensions shown in the diagram.

7. How many acres are there in the whole tract of land?

For convenience in raising different kinds of vegetables, the farmer divided his farm into plots or small fields with the dimensions as indicated in the diagram.

8. How many acres are there in each of the large squares?

9. How many acres are there in each of the small squares?

10. Each of the narrow rectangular strips contains how many acres? Show two ways of finding the area of one of these rectangular plots.

11. Mr. Dorsey's farm is 160 rods long and 80 rods wide. How many acres does it contain?

12. How many acres are there in a rectangular field a quarter of a mile long and a half of a quarter of a mile wide?

13. The deed to a certain farm states that the title to the east half of the southwest quarter of section 12 in a certain township has been transferred to the owner. How many acres are there in this farm? How much did the farm cost at $125 an acre?

14. Draw a diagram of a township and locate the portion of the twelfth section described in Problem 13.

15. How many acres are there in a garden plot 330 feet long and 165 feet wide?

16. How many acres are there in a truck field 40 rods long and 32 rods wide?

Exercise 14. Gardening

1. Harry had a plot of potatoes 21 feet by 260 feet. What part of an acre was his plot?

2. He raised 24 bushels of potatoes and sold them at $1.75 per bushel. His expenses were $1.85. What was his profit? Find the profit per acre at the same rate.

3. Mable had a garden plot 165 feet by 33 feet. What part of an acre was her plot?

4. She raised onions and sold them as follows: August 31, 6 bags at 90 cents; October 4, 10 bags at $1.60; November 10, 15 bags at $2.00; November 27, 5 bags at $2.20. What were her total receipts? At the same rate what would have been the receipts of an acre of ground?

5. Henry cultivated 495 square rods of corn. How many acres did he have? His net profit was $111.03. Find his average net profit per acre.

6. Elizabeth cultivated a plot of corn 20 rods by 8 rods. What was the size of her plot in acres? Her net profit amounted to $34.35. Her receipts were $45.35. Find her expenses.

7. Arthur grew pickle onions in a garden 33 feet by 132 feet. What fraction of an acre did he cultivate? His receipts were $67.67. His expenses were $18.25. At this rate what would have been his net profit on one acre of ground?

8. Hilda cultivated 1.2 square rods of flowers. Her receipts were $14.95. Find her receipts per square rod. Find the receipts per acre at that rate.

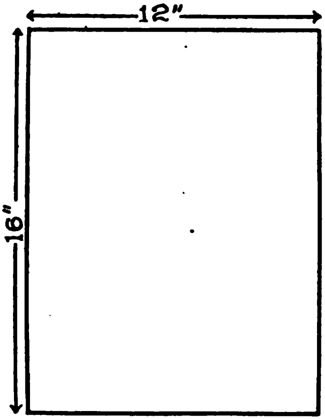
9. Mildred cultivated a plot 42 feet by $35\frac{1}{2}$ feet and cleared $12.12. Had she cultivated a whole acre with equal success, what profit would she have made?

10. Herman made a profit of $20.02 on a garden plot 77 feet by 29 feet. What would have been his profit on an acre of ground at the same rate?

Exercise 15. Scale Drawings

The maps in our geographies, the working drawings in our manual training work, the drawings for models of machines and the house plans for our architects are all drawn to a scale.

When we wish to make a reduced drawing of any object, we must decide on a scale that will enable us to get the drawing on the space which we have allowed for it.



SCALE 1" = 8"

1. Make a scale drawing of a rectangle 16 inches long and 12 inches wide. We cannot draw this rectangle on a page of this book, so we must decide upon a scale.

If we take a scale of 1" on the drawing $= 8''$ on the rectangle, how long will the drawing be? Using the same scale, how wide will the drawing be?

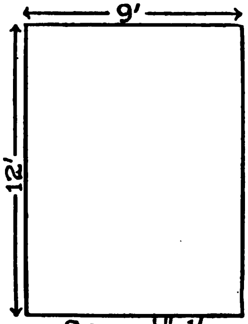
The drawing at the left represents the rectangle drawn to the scale of $1'' = 8''$.

2. Make a drawing, to a scale, of a garden plot 12 feet long and 9 feet wide.

The garden plot is larger than the rectangle, hence we must use a smaller scale.

The scale drawing at the right uses the scale $\frac{1}{8}'' = 1'$. If $\frac{1}{8}''$ represents $1'$ on the garden, how long should the drawing be made? How wide?

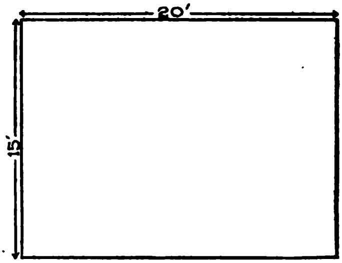
3. The width of the drawing is what fraction of the real width of the garden?



SCALE $\frac{1}{8}'' = 1'$

4. Draw a plan of the garden plot described in Problem 2 to the scale of $\frac{1}{4}'' = 1'$. Express this scale in a fractional form.

5. Draw on paper a plan of one of the blackboards in your room to a scale of $\frac{1}{4}'' = 1'$.



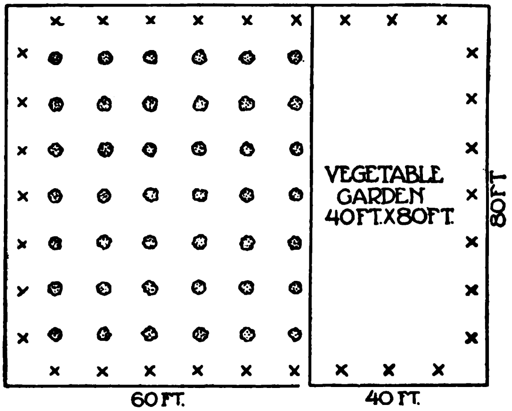
6. The drawing at the left represents the floor plan of a garage. Measure the width of this drawing and state the scale to which it was drawn. ($1'' = ?$ feet.)

7. How much was the cost of putting a concrete floor in this garage at 15 cents per square foot?

8. Make a drawing to a scale of a rug 9 feet by 12 feet.

9. Draw a plan of your school room to a scale of $\frac{1}{4}'' = 1'$, locating accurately the teacher's and pupils' desks.

This drawing[1] represents a fruit and vegetable garden.



10. Measure the lines in the drawing and by means of the dimensions that are given find the scale of the drawing.

11. By means of the scale ($1'' = 40'$) find the length and width of the fruit garden. Give the dimensions of the vegetable garden.

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[1] From a Bulletin issued by the U. S. Department of Agriculture.

12. The crosses represent grape vines set around the outside of the whole tract. By means of the scale find how far apart these grape vines are set.

13. How far apart are the rows of trees in the fruit garden? How far apart are the trees in each row?

Measure from the center of one tree to the center of another.

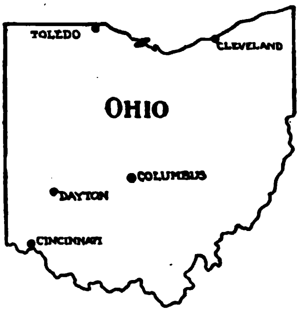
14. Draw a plan of a baseball diamond, using a scale of $\frac{1}{4}''$ to the foot.

15. Draw a plan of your home and yard, using $\frac{1}{8}''$ to a foot.

Exercise 16. Maps

The map of Ohio is drawn to a scale of $1'' = 150$ mile.

1. Measure the distance from Cleveland to Toledo on your ruler. Then use the scale of $1'' = 150$ miles and compute the distance in a straight line from Toledo to Cleveland. Check your measurements by the distance given in the table following Problem 3.



Scale 1″=150 Mi.

2. In the same way find the distances from Cleveland to Columbus; Columbus to Dayton; Toledo to Cincinnati; Cleveland to Cincinnati and Cincinnati to Columbus.

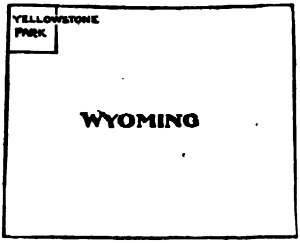
The distance by rail between any of these cities will be longer than the distance measured in a straight line.

3. How wide is the state of Ohio from Indiana line to the Pennsylvania line?

Approximate Distances

Toledo to Cleveland 95 miles.
Cleveland to Columbus 125 miles.
Columbus to Dayton 65 miles.
Toledo to Cincinnati 180 miles.
Cleveland to Cincinnati 215 miles.
Cincinnati to Columbus 95 miles.

4. Use the map of your state in the same way to find the distances between the most important cities.



Scale 1″=230 Mi.

5. The map at the left represents the state of Wyoming drawn to a scale of 1″ = 230 miles. By using this scale, find the length and width of this state in miles.

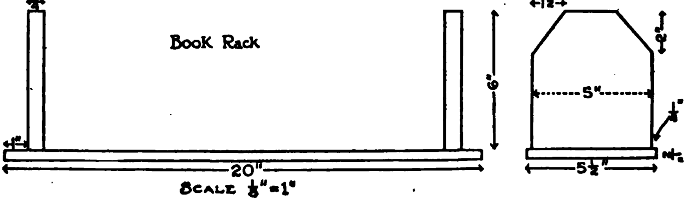
6. Find the area of Wyoming in square miles, using the length and width found in Problem 5. Compare this area with the correct area which you can find in your geography.

7. Find the approximate area in square miles of Yellowstone National Park in the northwest corner of the state.

Exercise 17. Shop Drawings

Before making an article in the shop one usually makes a working drawing to serve as a guide. These drawings are usually made to a scale in order to give the correct proportions of the various parts and thus enable the worker to see that his design is properly proportioned.

The working drawing of a book rack, shown below, is drawn to the scale of $\frac{1}{8}'' = 1''$.



There are two views in this drawing, a side view and an end view. The length of the base and the thickness of the ends are shown in the side view, and the width of the ends and the width of the base are shown in the end view.

1. From the drawing find the length, width and thickness of the baseboard.

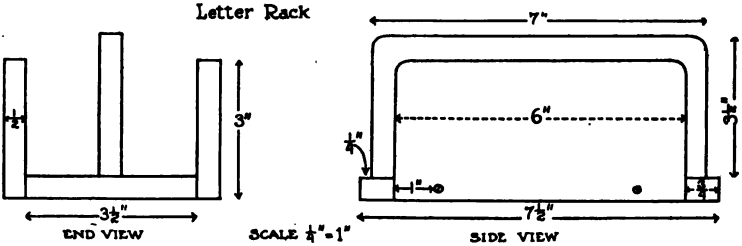
2. Find the height, width and thickness of an upright end.

3. How far from each end of the baseboard is each of the upright ends placed?

4. Show how to mark off the corners of the end boards.

5. From the dimensions given in the drawing find the number of board feet of lumber in this book rack.

Count the end pieces before the corners are sawed off. Remember that ½-inch material is figured as an inch in thickness in estimating the board feet in a piece of lumber.



6. To what scale is the letter rack in the above illustration drawn?

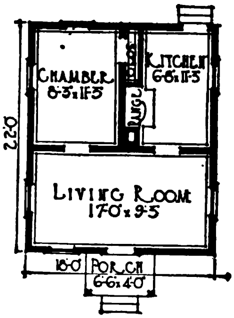
7. What views of the letter rack are shown?

8. How many pieces of wood are needed to make the letter rack?

9. From the drawing, give the dimensions of each piece.

10. Find the number of board feet of lumber in the letter rack.

Exercise 18. House Plans



This illustration shows a reproduction of a floor plan of a small summer cottage that was drawn to a scale by an architect.

1. Measure the width of the drawing with your ruler. How wide is it? From the plan, find the width of the cottage. This plan in the book is then drawn to a scale of 1″ = ? ft.

2. How many rooms are there in this cottage? Give the dimensions of each room.

3. How many windows are there in the plan? By means of the scale, find the width of the windows.

4. How many doors are there in the plan? Find the width of each of the doors.

5. Draw a floor plan for a 4-room cottage of your own design. Use the same means of indicating the various parts of the drawing as shown in the above illustration.

Exercise 19. Concrete Work

1. What is the cost of laying a cement walk 50 feet long and 5 feet wide at $12\frac{1}{2}$ cents a square foot?

2. Estimate the cost of cementing the floor and side walls of a cellar 20′ by $16\frac{1}{4}$′ by 8′ at $12\frac{1}{2}$ cents a square foot.

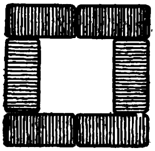
3. A concrete foundation 3 feet wide and 8 feet deep was placed under a building with outside measurements of 120′ by 100′. How many cubic yards of concrete were used?

Suggestion: Do not count the corners twice. Show that there are 2 walls 120 feet long and that the two ends between them must be considered 94 feet long on account of the side walls being 3 feet thick.

Exercise 20. Brick Work

1. A brick wall 40′ by 1′ by 8′ contains how many bricks, counting 22 bricks to 1 cubic foot of the wall?

2. If it takes 1 barrel of lime and $\frac{5}{8}$ cubic yard of sand to make the mortar for 1000 bricks, how much of each is needed to lay the wall described in Problem 1?



3. How many bricks are required for a chimney 27 feet high and built as shown in the diagram at the left, counting 5 bricks to a foot in height?

4. How much would the bricks for this chimney cost at $12 per M?

Exercise 21. Roofing

The unit for measuring roof surface is the "square." A "square" is 10 feet long and 10 feet wide. How many square feet does it contain?

1. Shingles average 4 inches in width. If a shingle is exposed 4 inches to the weather, how many square inches of surface will it cover?

2. How many square inches of surface are there in a "square" (100 square feet)? At that rate how many shingles will be needed to cover a square?

The first row of shingles must be laid double and some are broken or otherwise defective. For this reason, carpenters usually count **1000** *shingles to a square* when they are exposed 4 inches to the weather.

3. The roof of a tool shed in a school garden is 18½′ by 8′ on each side. How many shingles should be ordered for putting a shingle roof on it? How much would the shingles cost at $5.50 per M?

CHAPTER IV

PERCENTAGE

The term *per cent* is widely used in business. It is a short form by which we can express the rate of gain in business investments, interest on money, discounts on bills of goods, the proportion of butter fat in milk, the rate of insuring our property against loss by fire, and the rate of commission received for selling goods.

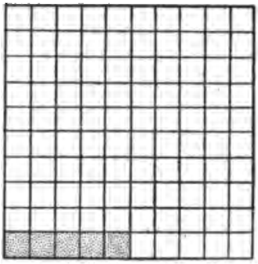
The expression *per cent* means *hundredths.* When one borrows money and agrees to pay 6 per cent of the amount each year for the use of the money, it means that he must pay 6 *hundredths* (or $\frac{6}{100}$) of that amount for the use of the money. When we say that 4 per cent of milk is butter fat, we mean that $\frac{4}{100}$ of the milk is butter fat. In other words, in every 100 pounds of such milk, there are 4 pounds of butter fat.

Instead of saying that a man gains one-tenth on the money he has invested, we say that he gains ten per cent on his money, meaning that he gains ten dollars on every hundred dollars that he has invested.

Outside the business world, percentage is much employed in scientific and statistical reports.

Instead of writing the words *per cent,* we may save time by using the sign %.

Exercise 1



1. In the diagram at the right how many small squares are there in the large square? How many small squares are shaded?

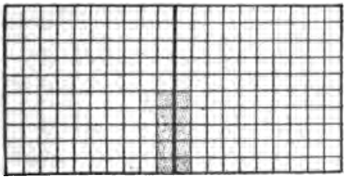
2. The shaded portion represents how many hundredths of the large square?

Since *per cent* means *hundredths*, $\frac{5}{100}$ of the large square may be expressed as 5% of the large square. The shaded portion then, is what per cent of the large square?

3. Draw a diagram on squared paper to show 2% of a large square; 8% of such a square, and 10% of a square.

4. Draw diagrams on squared paper to show the meanings of 20%; of 25%; of 50%; of 75%.

5. What is 5% of 200?



The number 200 may be represented by 200 small squares in the shape of a rectangle. The heavy line in the middle divides the rectangle into two parts, each containing 100 squares. 5% of 100 of these squares has aready been shown to be 5 squares. The shaded portion of the rectangle represents 5% of 200 squares, or 10 squares.

6. Draw a diagram on squared paper to show 8% of 200.

7. Draw diagrams on squared paper to represent 6% of 400; 3% of 300; 25% of 200.

Exercise 2. Per Cents Expressed as Decimals

5% has the same value as the common fraction $\frac{5}{100}$. But the common fraction $\frac{5}{100}$ may also be expressed as a decimal, .05.

Then $5\% = \frac{5}{100} = .05$.

Express the following per cents as common fractions, then as decimals as shown in the preceding form:

1. 2 per cent.
2. 6 per cent.
3. 10 per cent.
4. 25 per cent.
5. 3%.
6. 1%.
7. 8%.
8. 20%.
9. 50%.
10. 75%.
11. 40%.
12. 4%.
13. 12½%.
14. 33⅓%.
15. 37½%.
16. 16⅔%.

17. What is 6% of $300? 6% of $300[1] = .06 of $300 = ?

18. What is 12½% of $40?

12½% = .12½ or .125. 12½% of $40 = .125 of $40 = ?

Find the following:

19. 4% of $150.
20. 10% of 75 pounds.
21. 1% of 200 bushels.
22. 50% of $18.
23. 20% of $400.
24. 16⅔% of $48.
25. 8% of 2000 pounds.
26. 6% of $500.
27. 37½% of 40 pounds.
28. 75% of 1200 bushels.
29. 11% of $3750.
30. 3% of $126.50.
31. 2% of $26.40.
32. 40% of 800 bushels.
33. 55% of 2000 bushels.
34. 33⅓% of 210 pounds.
35. 20% of 45 bushels.
36. 5% of 360 eggs.
37. 9% of $125.
38. 40% of $0.80.
39. 7% of $250.
40. 50% of 640 bushels.
41. 25% of 950 bushels.
42. 3% of $85.60.

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[1] One Per Cent Method: Teachers who prefer the one per cent method may substitute the following explanation: $1\% \text{ of } \$300 = \frac{1}{100} \text{ of } \$300 = \$3.00$. $6\% = 6 \times \$3.00 = \$18.00$.

Exercise 3

1. Out of every ream of 500 sheets, 4% is allowed for waste in printing. How many sheets are allowed for waste in each ream?

2. The attendance at a basket ball game was 215. The high-school students composed 60% of the audience. How many high-school students attended the game?

3. A sixth grade cleared $45.20 on a school entertainment. They spent 75% of this amount for pictures for their school room. How much did the pictures cost?

4. A stenographer saves 10% of her salary of $12 per week. How much does she save each week? At that rate how much can she save in 50 weeks?

5. Of the books in a city library, 60% are story books. How many story books are there in this library of 8500 volumes?

6. The historical section of that library forms 10% of the total number of books. How many historical books are there in the library?

7. A store handling athletic goods sells footballs costing $3.00 at a profit of 40%. How much do they make on 4 such sales?

8. In a certain sixth-grade class of 35 pupils, 60% are girls. How many girls are there in the class? How many boys?

9. In a country school, 20% of the pupils are in the first grade. If the enrollment of the school is 25 pupils, how many pupils are there in the first grade?

10. A book that costs 40 cents is sold at a gain of 25%. What is the selling price of the book?

11. Martha raised 75 chickens. She sold 80% of them and kept the rest of them. How many did she sell? How many did she have left?

12. A hardware dealer secured a reduction of 2% on a bill of goods amounting to $175, by paying cash. What was the amount of his reduction for cash payment?

13. An 8-pound roast lost 25% of its weight when roasted. What was the weight of the roast after being roasted?

14. Mr. Dunn has 40 trees in his orchard. 40% of them are apple; 30% are peach; 20% are cherry and 10% are pear. Find the number of each kind.

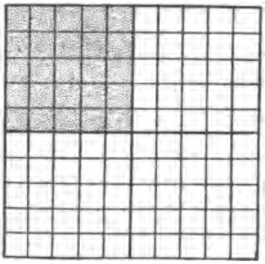
15. A real estate dealer sold a house and lot costing $2400 at a gain of 15%. How much did he gain? Find the selling price of the property.

16. A farmer, owning 320 acres of land, has 40% in crops, 5% in an orchard, 15% in timber and the rest in pasture. How many acres does he have in pasture?

17. James had 350 marbles and gave his brother 20% of them. How many marbles did James have left?

18. On a stormy day, 25% of the pupils in a country school, with an enrollment of 20 pupils, were absent. How many pupils were present?

Exercise 4. Equivalent Per Cents of Fractions



1. How many small squares are there in the whole figure at the left?

2. What part of the whole figure is the shaded portion?

3. How many small squares are there in the shaded portion?

4. The shaded portion is how many hundredths of the whole figure? How many per cent?

5. Then $\frac{1}{4}$ of the figure $=$? % of the figure?

6. By means of the diagram find how many per cent of the figure are equal to $\frac{1}{2}$ of the figure?

7. $\frac{3}{4}$ of the figure (the unshaded portion) $=$? % of the figure?

From the preceding question we have found that:

$$\frac{1}{4}=25\%;\ \frac{1}{2}=50\% \text{ and } \frac{3}{4}=75\%.$$

8. What per cent of a number is $\frac{3}{5}$ of it?

Any number is equal to $\frac{100}{100}$ or 100% of itself. $\frac{3}{5}$ of a number $=\frac{3}{5}$ of 100% or 60% of the number.

9. What per cent of a number is $\frac{1}{3}$ of it?

The number $=$ 100% of itself. $\frac{1}{3}$ of the number $=\frac{1}{3}$ of 100% or $33\frac{1}{3}\%$ of the number.

Find the per cents equal to the following fractional parts of numbers:

10. $\frac{1}{2}$
11. $\frac{1}{4}$
12. $\frac{2}{4}$
13. $\frac{3}{4}$
14. $\frac{1}{3}$
15. $\frac{2}{3}$
16. $\frac{1}{5}$
17. $\frac{2}{5}$
18. $\frac{3}{5}$
19. $\frac{4}{5}$
20. $\frac{1}{6}$
21. $\frac{1}{7}$
22. $\frac{1}{8}$
23. $\frac{3}{8}$
24. $\frac{5}{8}$
25. $\frac{7}{8}$
26. $\frac{1}{9}$
27. $\frac{1}{10}$
28. $\frac{3}{10}$
29. $\frac{7}{10}$

Daily drill should be given until the pupils can readily give the equivalent per cents for all the fractions given in this exercise. Pupils should be shown how these facts will enable them to use short methods in solving such problems as those in Exercise 6.

Exercise 5. Fractional Equivalents of Per Cents

Not only do we need to know that $\frac{5}{8}=62\frac{1}{2}\%$, but we must also be able to see that $62\frac{1}{2}\%=\frac{5}{8}$.

1. What fraction is equal to 25%?

$25\%=\frac{25}{100}$, which reduced to its lowest terms $=\frac{1}{4}$.

2. What fraction is equal to $37\frac{1}{2}\%$?

$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100}$. $\frac{37\frac{1}{2}}{100}$ means $37\frac{1}{2} \div 100$. $\frac{75}{2} \div \frac{100}{1} = \frac{\overset{3}{\cancel{75}}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{3}{8}$.

In the same manner find the fractions equal to the following per cents:

3. 50%.
4. 75%.
5. $33\frac{1}{3}\%$.
6. $66\frac{2}{3}\%$.
7. 20%.
8. 40%.
9. 60%.
10. 80%.
11. $16\frac{2}{3}\%$.
12. $14\frac{2}{7}\%$.
13. $12\frac{1}{2}\%$.
14. $62\frac{1}{2}\%$.
15. $87\frac{1}{2}\%$.
16. $11\frac{1}{9}\%$.
17. 10%.
18. 30%.

Exercise 6. Using Fractional Equivalents

1. What is 25% of $200?

25% = $\frac{1}{4}$. Then 25% of $200 = $\frac{1}{4}$ of $200 = $50.

2. What is $33\frac{1}{3}\%$ of $75?

3. What is $66\frac{2}{3}\%$ of 210 pounds?

4. What is 20% of $15?

Find the following per cents.

5. 40% of $24.
6. $16\frac{2}{3}\%$ of 24 cents.
7. $37\frac{1}{2}\%$ of 32 pounds.
8. 10% of $500.
9. $87\frac{1}{2}\%$ of 80 pounds.
10. 50% of $400.
11. 60% of 480 bushels.
12. 70% of 160 pounds.
13. 75% of 60 bushels.
14. $62\frac{1}{2}\%$ of 160 acres.
15. 80% of $120.
16. $12\frac{1}{2}\%$ of $64.
17. 10% of $90.
18. 25% of $16.
19. $33\frac{1}{3}\%$ of 240 pounds.
20. 60% of 1500 pounds.

21. A farmer planted 40% of his farm of 80 acres in corn How many acres of corn did he have?

Exercise 7

1. A hog loses about $\frac{1}{3}$ of its weight when butchered. What per cent of the weight of the hog is lost when it is dressed for meat?

2. A hog, which weighs 204 pounds when alive, will weigh how much when dressed? (Count the shrinkage as $33\frac{1}{3}\%$ of the live weight.)

3. Roy sold his bicycle for 50% of its cost. If he paid $24.50 for it, how much did he receive for it?

4. A real estate agent sold a farm costing $8000 at a gain of 25%. How much was his profit? How much did he receive for the farm?

5. In a certain school, pupils whose grades are above 90 are excused from the final examination. 40% of the pupils in a class of 30 were excused from the examination. How many pupils were excused? How many were required to take the examination?

6. Of the 18 pupils taking the final examination, $16\frac{2}{3}\%$ made grades above 90. How many made grades above 90?

7. A farmer raised 1640 bushels of corn. He kept $37\frac{1}{2}\%$ of it for his own use. How many bushels did he keep for his own use?

8. By applying fertilizers a farmer increased his cotton crop $62\frac{1}{2}\%$. If his yield was 480 pounds per acre before he applied the fertilizers, how many pounds did he raise per acre after applying the fertilizers?

9. About $12\frac{1}{2}\%$ of corn meal is water. How many pounds of water could be obtained from 56 pounds of corn meal?

10. About 50% of the weight of a squash is refuse material. How many pounds of edible material are there in a squash weighing $4\frac{1}{2}$ pounds?

Exercise 8. Dairy Problems

When cream is churned, the particles of fat collect together and form butter. The milk of some cows contains more butter fat than that of others. By means of a test, called the Babcock test, the per cent of butter fat can be found.

1. A cow gives 24 pounds of milk per day on the average. Her milk test shows 4.1% of butter fat. How many pounds of butter fat does she produce per week? (4.1% = .041)

2. Another cow in the same herd gives 28 pounds of milk per day, but her milk tests only 3.7% of butter fat. Find the amount of butter fat produced by this cow in a week.

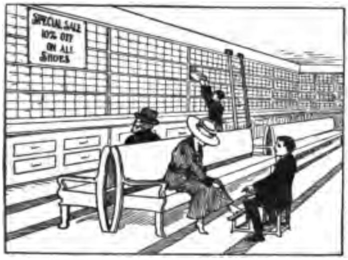
3. Mr. Werner's cow tests 4% butter fat and Mr. Orr's cow tests 4.1%. Each cow gives about 25 pounds of milk a day. When butter fat is selling at 36 cents a pound, which cow will yield the greater weekly profit from butter fat and how much?

4. One of the cows tested by John Dorwaldt gave, on an average, 26 pounds of milk a day during January. Her milk tested 3.6% butter fat. What was the value of this butter fat at 32 cents a pound?

5. A dairyman shipped to a butter factory a can of cream weighing 65 pounds. This cream tested 35% butter fat. How many pounds of butter fat were there in the can? How much was the butter fat worth at 35 cents a pound?

6. The next day he churned 65 pounds of cream which tested 35% butter fat. How many pounds of butter fat were there in that amount of cream? The amount of butter obtained was 17% more than the number of pounds of butter fat. How much was the butter worth at 32 cents a pound?

Exercise 9. Special Sales



Some merchants have special sales in order to close out certain lines of goods or to increase their sales during dull times. They advertise their goods at a reduction of a certain per cent from the regular price.

1. A camera marked $9.00 is listed in a special sale at 25% off. What is the sale price of the camera?

$25\% = \frac{1}{4}$. $\frac{1}{4}$ of \$9.00 = \$2.25. \$9.00—\$2.25 = \$6.75, the sale price.

2. A suit case marked $8.00 was sold at a special sale at a discount of 20%. What was the selling price of the suit case?

3. A tennis racket marked $3.00 was sold at a reduction of 35%. If the racket cost the dealer $1.75, what was his profit at the sale price?

4. A shoe dealer advertised a reduction of 10% on the prices of all of his shoes. How much was the sale price on a pair of shoes which he had formerly sold at $5.00 per pair?

5. A furniture dealer sold a $24 table at a reduction of 20%. How much did he receive for the table?

6. Elizabeth's father bought a piano priced at $450 at a reduction of $33\frac{1}{3}\%$. How much did he pay for it?

7. Roger bought a bicycle marked $24.00 at a reduction of 10% in price. How much was the price reduced? How much did he pay for the bicycle?

8. John joined a company of Boy Scouts. His father sold the boys their equipment at 20% off his regular prices. If they purchased an outfit which amounted to $48 at regular prices, how much did they have to pay for it?

Exercise 10. Finding What Per Cent One Number Is of Another

1. A ball team won 3 out of a series of 5 games. What per cent of the games did it win?

The team won 3 out of 5, or $\frac{3}{5}$ of the games played. $\frac{3}{5}=60\%$. Therefore the team won 60% of the games.

2. A merchant sold a sweater that cost $3.00 for $4.50. What was his per cent of gain?

$4.50—$3.00 = $1.50. $1.50 = $\frac{1}{2}$ or 50% of the cost, $3.00.

3. Roller skates costing $1.25 were sold for $1.50. What was the per cent of gain?

4. Packages of breakfast food costing 12 cents each are sold by a grocer for 15 cents each. Find his per cent of gain.

5. A basket ball team won 12 out of 16 games. What per cent of the games played did they win?

6. Fred played in 10 ball games one season. He was "at bat" 48 times and made 16 safe hits. His safe hits were what per cent of the number of times he went to bat?

7. A clerk received a salary of $15 per week. He put $3 of this amount in his savings account each week. What per cent of his salary did he save?

8. A truck farmer raised 400 bushels of potatoes and kept 40 bushels for seed and home consumption. The amount kept for his own use was what per cent of the amount that he raised?

9. An arithmetic test contains 10 problems of practically the same difficulty. What should be Daniel's per cent grade if he solved 8 of them correctly?

10. What grade should Eleanor receive if she solved 9 problems of the test correctly?

11. A pint is what part of a quart? What per cent of a quart?

12. A foot is what per cent of a yard?

13. A peck is what per cent of a bushel?

14. A pint is what per cent of a gallon?

15. What per cent of a pound are 8 ounces? 4 ounces? 6 ounces?

16. If you earn $1.00 and spend 25 cents, what per cent of the dollar do you spend?

17. A furniture dealer sold a dressing table marked $20 for $16. What was the amount of the reduction in price? The reduction was what per cent of the marked price?

Exercise 11

1. 39 is what per cent of 104?

$$\begin{array}{r} .37\frac{1}{2} \\ 104\overline{)39.00} \\ \underline{31\ 2} \\ 7\ 80 \\ \underline{7\ 28} \\ \frac{52}{104}=\frac{1}{2} \end{array}$$

Explanation[1]: 39 is $\frac{39}{104}$ of 104. To change a common fraction to a decimal we divide the numerator by the denominator. Carrying the quotient out to hundredths, $39 \div 104 = .37\frac{1}{2}$. . $.37\frac{1}{2} = 37\frac{1}{2}$ hundredths or $37\frac{1}{2}\%$. Then 39 is $37\frac{1}{2}\%$ of 104.

$37\frac{1}{2}\%$ may also be expressed as 37.5% by changing the fraction of a per cent to a decimal.

2. 26 is what per cent of 140?

3. 23 is what per cent of 207?

4. $9 is what per cent of $150?

5. 27 bushels is what per cent of 72 bushels?

6. $44 is what per cent of $400?

7. $4.62 is what per cent of $84?

8. $.75 is what per cent of $37.50?

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[1] The **One Per Cent Method** may be used if the teacher prefers.

1% of 104 = 1.04. 39 is as many per cent of 104 as 1.04 is contained times in 39, which = $37\frac{1}{2}$. Therefore, 39 is $37\frac{1}{2}\%$ of 104.

9. 24 is what per cent of 50?

10. $31.25 is what per cent of $250?

11. A newsboy bought 50 newspapers and sold 44 of them. What per cent of his papers did he sell? (44 is what per cent of 50?)

12. Roller skates costing $1.25 per pair were sold by one dealer at $1.75 per pair. What was his per cent of profit?

First find the gain. The gain is what per cent of the cost?

13. In a spelling test of 50 words, Ruth spelled 46 words correctly. What per cent of the list did she spell correctly?

14. A liveryman bought a horse for $120 and after training him sold him for $160. What was his per cent of gain?

15. A laborer works 8 hours a day. What per cent of a day does he work?

16. John raised 72 chickens and sold 45 of them. What per cent of his chickens did he sell? What per cent of them did he have left?

17. Ellen found a nest containing 28 eggs. When she candled them, she found that 4 of them were spoiled. What per cent of the eggs were not good?

18. Mr. Waterman butchered a hog weighing 243 pounds. He found that it weighed only 162 pounds when dressed. What per cent of its weight was lost in butchering?

19. A lawyer received a fee of $40 for collecting a bill for $800. What per cent of the amount of the bill did he receive for collecting?

20. A truck farmer used 1 acre of his 20-acre farm for his house and grounds. What per cent of his farm did he use for his house and grounds? How many per cent of his farm did he have left for garden?

Exercise 12

1. 40 is $33\frac{1}{3}\%$ of what number?

$33\frac{1}{3}\% = \frac{1}{3}$. If 40 is $\frac{1}{3}$ of the number, $\frac{3}{3}$ of the number $= 3 \times 40 = 120$, the number.

2. 24 is 80% of what number?

$80\% = \frac{4}{5}$. If $24 = 4$ fifths of the number one fifth of the number $= \frac{1}{4}$ of 24, or 6. $\frac{5}{5}$ of the number $= 5 \times 6 = 30$, the number.

3. 48 is $37\frac{1}{2}\%$ of what number?

4. 16 is 20% of what number?

5. 64 is $66\frac{2}{3}\%$ of what number?

6. Mrs. Ward sold 40 chickens, which was $33\frac{1}{3}\%$ of the number that she raised. How many did she raise?

7. Two boys wished to know what per cent of the weight of alfalfa hay came from the leaves. They took 20 pounds of alfalfa and carefully picked off all the leaves. They weighed them and found that the leaves weighed about 9 pounds. The leaves form what per cent of the weight of the hay?

8. A dealer bought books for 80% off their list price. What was the list price if he paid $1.20 for them?

9. A farmer sold a horse for $112 at a loss of 20% of the cost. How much did the horse cost him.[1]

The selling price of the horse = 100% of the cost—20% of the cost = 80% of the cost. $112 is 80% of what amount?

10. A real estate dealer sold a house for $2750 at a gain of 10% of the cost. How much did the house cost him?

$2750 = how many per cent of the cost, since he **gained** 10%?

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[1] **One Per Cent Method:** 80% of cost = $112. 1% of cost = $\frac{1}{80}$ of $112 = $1.40. 100% of the cost = 100 × $1.40, or $140, the cost of the horse.

This method may be substituted for the fractional method if the teacher prefers. The two methods should not be mixed or the pupils will become confused.

Exercise 13. Interest

If we rent an auto truck for \$2 an hour, we pay that amount for the benefits which we expect to receive from the use of it. If, however, we borrow \$1000 from a friend and buy an auto truck with it, we also pay a him a certain amount for the use of this money because it enables us to receive certain benefits.

Money which is paid a lender for the use of money is called *interest.*

Interest is computed as a certain per cent of the amount loaned for a year's use of the money. For example: 6% interest on \$300 means that 6% or 6 hundredths of \$300 will be paid each year for the use of that amount of money.

1. Find the interest on \$500 for 1 year at 6%.

6% = .06. 6% of \$500 = .06 × \$500 = \$30.00, the interest.

Find the interest on the following amounts for 1 year:

2. \$150 at 3%.
3. \$200 at 6%.
4. \$375 at 6%.
5. \$125 at 7%.
6. \$290 at 6%.
7. \$48 at 3%.
8. \$3000 at 5%.
9. \$5000 at 5%.
10. \$1500 at $5\frac{1}{2}$%.

11. Find the interest on \$200 for 6 months at 6%.

6% of \$200 = .06 × \$200 = \$12.00, the interest for 1 year. 6 months = $\frac{1}{2}$ year. Then the interest for 6 months = $\frac{1}{2}$ of \$12.00 = \$6.00.

Find the interest on the following amounts:

12. \$500 for 6 months at 6%.
13. \$250 for 4 months at 6%.
14. \$700 for 3 months at 6%.
15. \$1800 for 6 months at 5%.
16. \$200 for 5 months at 7%.
17. \$150 for 8 months at 6%.
18. \$2400 for 8 months at 5%.
19. \$3600 for 9 months at 5%.
20. \$1000 for 5 months at 6%.
21. \$4000 for 2 months at 8%.
22. \$2000 for 9 months at 6%.
23. \$1600 for 2 months at 10%.

24. Find the interest on $400 for 1 year 8 months at 6%.

	$ 400	
	.06	
	24.00	interest for 1 year.
6 months = $\frac{1}{2}$ year	12.00	interest for 6 months.
2 months = $\frac{1}{6}$ year	4.00	interest for 2 months.
	40.00	interest for 1 year 8 months.

25. Find the interest on $600 for 1 year 9 months at 6%.

26. What is the interest on $1000 for 1 year 10 months at 5%?

27. A farmer bought a farm for $9000. He paid $5000 cash and borrowed the rest of the money at 5% interest. How much interest did he have to pay each year?

28. A merchant borrowed $800 at 6% interest to buy holiday goods. How much interest did he owe at the end of 3 months? What was the total amount of money that he had to pay the lender at the end of that time?

Exercise 14. Supplementary Interest Problems

1. Find the interest on $250 for 1 year 10 months 18 days at 6%.

	$250	
	.06	
	$15.00	interest for 1 yr.
6 mo. = $\frac{1}{2}$ yr.	7.50	interest for 6 mo.
3 mo. = $\frac{1}{2}$ of 6 mo.	3.75	interest for 3 mo.
1 mo. = $\frac{1}{3}$ of 3 mo.	1.25	interest for 1 mo.
15 da. = $\frac{1}{2}$ of 1 mo.	.625	interest for 15 da.
3 da. = $\frac{1}{5}$ of 15 da.	.125	interest for 3 da.
	$28.250	interest for 1 yr. 10 mo. 18 da.

Find the interest on:

2. $350 for 1 year 8 months 15 days at 6%.
3. $120 for 1 year 7 months 10 days at 7%.
4. $640 for 2 years 5 months 20 days at 6%.
5. $2000 for 2 years 8 months 18 days at 5%.
6. $875 for 1 year 9 months 24 days at 6%.
7. $500 for 1 year 10 months 15 days at 7%.
8. $600 for 1 year 2 months 13 days at 6%.
9. $1500 for 1 year 8 months 10 days at 5%.
10. $2400 for 1 year 3 months 15 days at 5%.
11. $375 for 1 year 1 month 3 days at 6%.
12. $500 for 1 year 4 months 12 days at 6%.

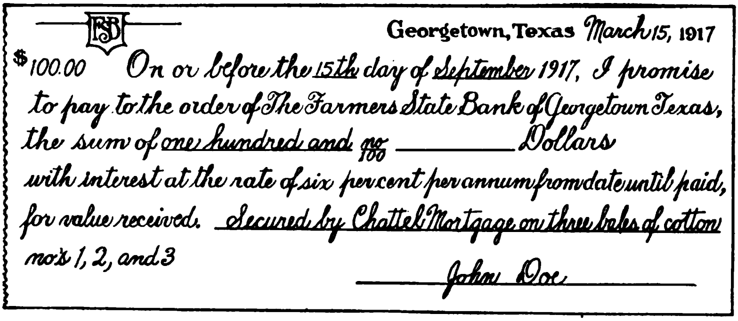
Exercise 15. Borrowing Money

When we borrow money, we are required to give a note to the lender stating the date, the amount of money borrowed, the time for which the money is loaned and the rate of interest that is to be paid on the note. If the lender has a great deal of confidence in the honesty of the borrower and his ability to pay back the money, he may merely require him to sign the note.

In order to make a loan more secure, the borrower is often required to get another party to sign the note with him. The second signer is said to be *security* for the payment of the note and is required to pay it if the borrower is unable to do so.

Mortgages are also given to secure the payment of notes. A mortgage is a written contract which gives the lender the right to sell property named in the mortgage if the borrower fails to pay a note.

The following note shows the form of a promissory note that is secured by a mortgage[1]:



Georgetown, Texas March 15, 1917

$100.00 On or before the 15th day of September 1917, I promise to pay to the order of The Farmers State Bank of Georgetown Texas, the sum of one hundred and no/100 ——— Dollars with interest at the rate of six per cent per annum from date until paid, for value received. Secured by Chattel Mortgage on three bales of cotton nos 1, 2, and 3

John Doe

The above note was given on the security of cotton that was to be raised that season. This is called borrowing on future crops. It enables the farmer to get sufficient money to meet his living expenses while he is raising his crop.

1. How many months will John Doe have to pay interest on this note?

2. Find the interest due on the note on September 15, 1917. What amount will John Doe owe the bank on that date?

3. Mr. Wade borrowed $1000 for 2 years, giving a mortgage on his farm as security. Find the interest on this amount for 2 years at 5½ per cent.

4. L. C. Henderson borrowed $500 from R. C. Danvers for 1 year at 6% interest. Write a note which Mr. Henderson would give to Mr. Danvers. Below Mr. Henderson's name sign A. M. Ward, who went on the note as security for Mr. Henderson.

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[1] The chattel mortgage on the cotton provided that the bank could sell the cotton at a public sale if John Doe failed to pay the note at the time specified. Any amount left after the note was paid was to be returned to John Doe.

5. How much interest will Mr. Henderson have to pay at the end of the year? What is the total amount of money which he will have to give Mr. Danvers when the note is due?

6. Find the interest on a note at its date of maturity for $275 for 3 months at 7 per cent. What will be the total amount due the lender?

7. What will be the interest on a loan for $250 made on a future corn crop at 10% for 8 months?

8. Find the amount due on a note at its date of maturity for $320 for 90 days (30 days count as a month) at 7%.

9. Write a note for $300, using fictitious names. What per cent of interest ought a loan of that amount to bear?

10. Find the interest on the note that you wrote in Problem 9. What amount would be due on the note at its maturity?

11. Edward loaned $57, the profit of his corn project, to his father on January 1, 1917. What amount will his father return to him on July 1, 1918, if the note bears simple interest at 6%?

12. A farmer borrowed $1500, giving a mortgage on his farm as security. What was his yearly interest payment at 6%?

13. John has a bank certificate for $105 bearing 3% interest. How much interest will he receive at the end of six months?

14. The First Liberty Bonds bore 3½% interest. How much was the yearly interest on a Liberty Bond for $50? For 100?

15. Find the interest on a note for $125 for 1 year 3 months at 6%. What will be the amount of this note when it is due?

16. A merchant borrowed $1250 for 3 months at 7% interest. Find the interest on this note. Find the amount of *the note*.

17. Find the interest on $1600 for 1 year 6 months at 6%.

Exercise 16. Savings Accounts

Savings Banks usually pay 3% or 4% interest on savings accounts. The interest is usually paid semi-annually (every 6 months) and is added to the savings account if the depositor does not withdraw the interest. No interest is paid on sums withdrawn between interest-paying dates. Each savings bank has certain rules about the payment of interest.

Pupils who have savings accounts should bring their bank books to the class, read the rules in regard to interest, and have the class compute the interest for the various amounts deposited.

1. Find the amount of a savings deposit of $80 made January 1, 1916, on January 1, 1917, if no deposits are made during that time. The interest is 3% per annum, payable semi-annually.

$80	deposit, January 1, 1916.
1.20	interest at 3% for 6 months.
$81.20	amount July 1, 1916.
1.22	interest at 3% for 6 months.
$82.42	amount due January 1, 1917.

The interest on $80 for 1 year at 3% is $2.40. For 6 months the interest is ½ of $2.40, or $1.20. This interest is added to the original deposit of $80, making $81.20. The depositor gets 3% interest on this amount from July 1, 1916 to January 1, 1917. The interest on $81.20 for 6 months at 3% = $1.22. $81.20 + $1.22 = $82.42, the amount due January 1, 1917. Most banks do not pay interest on fractional parts of a dollar. The interest on $81 for 3% for 6 months = $1.215 or $1.22, so in this case there would be no difference whether the fractional part of a dollar were counted or not.

2. Find the amount of interest on a savings account of $45 at 3% for 6 months.

3. John Millish saved $72 from his garden project and put this in a savings bank, October 1, 1916. If he makes no deposits before January 1, 1917, what amount will he have to his credit at that interest-bearing date, the interest being computed semi-annually at 3%?

4. Albert Evans deposited $25 in a savings bank on July 2; $10 on September 1; $16 on October 2 and $12 on November 1. He made no withdrawals. According to the rules of his bank, any deposits made in the first 5 days of any month bear interest beginning with the *first of the month.* Find the amount of interest, at 3%, to January 1, as follows:

$25 from July 1 to January 1 = ?
$10 from September 1 to January 1 = ?
$16 from October 1 to January 1 = ?
$12 from November 1 to January 1 = ?
Total interest due January 1 = ?

Savings banks usually follow the practice of subtracting any withdrawals from the amount on deposit at the last interest-bearing date. Note this point in solving the next problem.

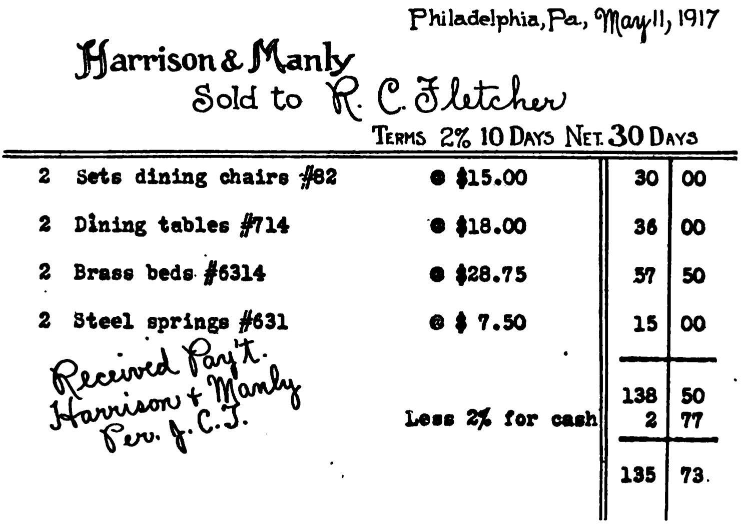
5. On July 1 Dorothy had $44 in her savings account. She deposited $16 on August 21 and $8 on October 1. On October 19 she withdrew $10 from her account. How much interest, at 3%, should she receive on January 1?

The withdrawal, $10, must be subtracted from her balance on July 1. $44—$10 = $34. She will get interest, then, on $34 from July 1 to January 1. The deposit of $16 on August 21 will not begin to draw interest until September 1 according to the method of counting only whole months.

6. Find the interest on the following savings account on July 1 at 3%: Balance, January 1, $32.25; Deposits: February 1, $5.50; April 2, $6.00; May 2, $3.00; Withdrawal, March 28, $4.00. Do not count fractional parts of a dollar (cents) in computing the interest.

7. Anna made $45.00 in raising poultry one year. She put this in a savings bank on January 1, 1916. How much will she have to her credit on January 1, 1918, if she does not make any deposits or withdrawals? Compute the interest at 3% semi-annually and do not count fractional parts of a dollar.

Exercise 17. Discounts



Philadelphia, Pa., May 11, 1917

Harrison & Manly

Sold to R. C. Fletcher

Terms 2% 10 Days Net 30 Days

2	Sets dining chairs #82	@ $15.00	30	00
2	Dining tables #714	@ $18.00	36	00
2	Brass beds #6314	@ $28.75	57	50
2	Steel springs #631	@ $ 7.50	15	00
			138	50
		Less 2% for cash	2	77
			135	73

Received Pay't.
Harrison + Manly
Per. J. C. J.

Most large firms allow a small reduction for prompt payment of a bill. In the illustration shown above, the terms are 2% reduction if the bill is paid in 10 days, or the full amount, or net, if the purchaser does not pay until after the expiration of the ten days. The reduction for cash or prompt payment in that bill amounted to $2.77. This reduction is called a *discount.*

Discounts are also allowed in reduction sales, which are common in large stores. The price that is placed on the article before it is discounted is called the *list price.*

1. A book dealer is allowed a discount of 20% from the list prices of books. How much does he pay for books listed at $1.25 each?

20% = $\frac{1}{5}$. $\frac{1}{5}$ of $1.25 = $.25. $1.25 the list price—$.25 "off" (or the discount) = $1.00, the cost of the book.

2. A clothing store advertises a special sale on women's coats, listed at $30, at a discount of $33\frac{1}{3}\%$. What is the sale price of the coats?

3. A hardware dealer secured a discount of 2% for prompt payment on a bill of goods amounting to $384.50. What was the net amount of the bill?

4. A school bought a piano from a local dealer priced at $350 at a discount of 20%. What did the piano cost the school?

5. A lumber dealer bought a bill of lumber amounting to $1240. He was allowed 3% discount for paying cash. How much was the amount of his discount? What was the net amount of his bill after deducting the discount?

6. Find the sale price at a special sale of *20% discount* on goods listed at the following prices:

	List Price	Discount	Sale Price
Silk Dress Goods	$2.50 per yard	?	?
Shoes	$5.00 per pair	?	?
Suits	$30.00	?	?
Suit Cases	$7.50	?	?
Umbrellas	$2.00	?	?
Dress Goods	$1.75	?	?
Ginghams	$.15 per yard	?	?

7. A druggist advertised $9.00 cameras at a discount of 15%. What was his sale price on the cameras?

8. In order to dispose of his supply of straw hats late in the season, a dealer advertised a 50% discount sale on all straw hats in his store. What was the sale price on a $4.00 hat; a $3.00 hat; a $5.00 hat?

9. A clothing store sold overcoats listed at $30.00 for $24.00. What was the per cent of discount on the coats?

10. A shoe store sold $6.00 Oxfords late in the summer for $4.00. What per cent discount did they give on the shoes?

CHAPTER V

EFFICIENCY ON THE FARM AND IN THE SCHOOL

Courtesy International Harvester Co.

In the early days of our country the people were few and land was plentiful. If one field began to lose its fertility, there was plenty of new land which could be used. Now most of the fertile tracts of land have been occupied, and over 100,000,000 people must be fed with the crops from our farms. It is therefore necessary that we learn how to retain the fertility of our soil and find the most profitable crops and methods of cultivation.

Exercise 1. The Value of Fertilizers

1. It is estimated that there is produced each year about 150,000,000 tons of barnyard fertilizer worth $200,000,000. What is the estimated value of a ton of this fertilizer?

2. Over $50,000,000 worth of this fertilizer is wasted each year. What per cent is this of the total value of the fertilizer?

Nitrogen is one of the most important food substances needed for growing good crops. Burning straw or cornstalks destroys the nitrogen.

3. The straw from a 25-bushel yield of wheat contains about 15 pounds of nitrogen. If a farmer raises 500 bushels of wheat, how many pounds of nitrogen are contained in the straw?

4. When purchased for fertilizing purposes nitrogen is worth about 20 cents a pound. Find the value of the nitrogen in the straw from the 500-bushel yield of wheat. If this straw is burned, how much loss will the farmer suffer?

5. A 50-bushel yield of corn contains about 25 pounds of nitrogen. How many pounds of nitrogen are contained in the corn crop from a 20-acre field yielding 50 bushels per acre?

6. If the farmer burns his cornstalks instead of plowing them under, and sells his corn, what is his loss in the fertilizing value of the nitrogen alone from the 20-acre field?

7. One hundred dairy cows weighing 1000 pounds apiece will produce 2,400,000 pounds of fertilizer in a year. How many tons of fertilizer do they produce per year? This fertilizer is worth over $1600. What is the value of the yearly production of fertilizer from a single cow?

Exercise 2. Rotation of Crops

At the Illinois Experiment Station an experiment on the value of crop rotation was conducted from 1888 to 1907. On one plot corn was grown every year; on a second plot corn and oats were grown in rotation (corn one year and oats the next); and on the third plot corn, oats, and clover were grown in rotation.

1. The yield on the plot on which corn was grown every year was 29 bushels per acre. What was the income from an acre of this corn if the average price was 40 cents a bushel?

2. The rent on this land and the cost of production were estimated at $10 per acre. What was the net profit per acre on the first plot?

3. The yield on the second plot, on which corn and oats were grown in rotation, was 48 bushels per acre. What was the income per acre from the corn on the second plot?

4. Estimating the cost of production to be the same, what was the net profit per acre from the corn on the second plot?

5. The yield on the third plot with a rotation of corn, oats and clover was 80 bushels per acre. Find the income at 40 cents per bushel from the corn on this plot. What was the net profit per acre?

Missouri Experiment. Rotation and Fertilizer

6. A similar experiment was conducted at the Missouri Experiment Station. On the plot where corn was grown every year the yield was only 11.8 bushels per acre. What was the income per acre on this land at 40 cents a bushel?

7. The yield on the plot where corn, wheat and clover were grown in rotation was 50.7 bushels per acre. What was the increase in the income per acre due to the crop rotation?

8. On a plot where fertilizer was used in addition to the crop rotation of corn, wheat and clover, there was a yield of 77.6 bushels per acre. Find the value of the increased production due to the use of the fertilizer, counting the corn at 40 cents a bushel.

Pennsylvania Experiment. Rotation and Fertilizer

In Pennsylvania a four-year rotation of corn, oats, wheat and hay was carried on for a period of 24 years on one plot without fertilizer and on another plot with fertilizer.

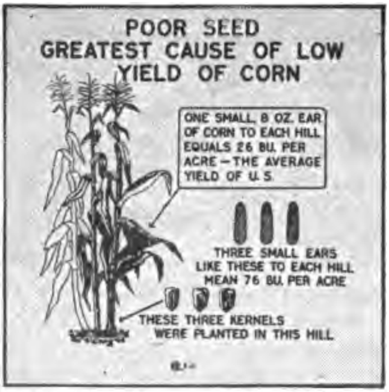
9. The average profit for the first 12 years on the plot without fertilizer was $10.58 per acre. During the second 12 years the profit was only $7.85 per acre. Find the total income from an acre for the period of 24 years.

10. The average profit for the first 12 years on which the fertilizer was used was $14.07 per acre and for the second 12 years $14.35 per acre. Find the total income from an acre of this plot for the period of 24 years.

11. Find the extra income for the 24 years due to the use of the fertilizer. If 12 tons of fertilizer were applied every 4 years, what was the profit per ton from the use of the fertilizer?

12. What benefit resulted from the use of the fertilizer in addition to the increased yield of the crops?

Exercise 3. Raising Corn


POOR SEED
GREATEST CAUSE OF LOW YIELD OF CORN
ONE SMALL, 8 OZ. EAR OF CORN TO EACH HILL EQUALS 26 BU. PER ACRE—THE AVERAGE YIELD OF U. S.
THREE SMALL EARS LIKE THESE TO EACH HILL MEAN 76 BU. PER ACRE
THESE THREE KERNELS WERE PLANTED IN THIS HILL

Courtesy International Harvester Co.

In 1913 the production of corn amounted to 2,700,000,000 bushels, worth $1,700,000,000. The value of the cotton crop in the same year was $800,000,000.

1. How much greater was the value of the corn crop than that of the cotton crop?

2. Corn is generally planted in hills $3\frac{1}{2}$ feet apart each way. How many square feet of ground are given to each hill? ($3\frac{1}{2} \times 3\frac{1}{2} = ?$)

3. There are 43,560 square feet in an acre. Show that there are approximately 3556 hills of corn per acre.

4. If 3 kernels are planted and each kernel grows and produces a stalk, how many stalks are there on an acre of corn?

5. If only one stalk in each hill grows a small ear weighing 8 ounces, what would be the yield per acre? (1 bushel of corn in the ear weighs 70 pounds. See Problem 3 for number of hills per acre.)

6. If two stalks in every hill each produced a 12-ounce ear, how many bushels would be produced per acre?

7. It is estimated that 12 per cent of the hills are missing in an average stand of corn. How many hills are missing per acre in an average stand? If these missing hills are due to poor seed corn, how much is the loss per acre due to poor seed, counting two 12-ounce ears to the hill?

8. Forty-five thousand ears of seed corn that were tested in Iowa gave an increased yield of 19½ bushels per acre. If the same methods had been employed and equal results secured on the 10,000,000 acres planted in that state, what would have been the increase in the corn crop in the state of Iowa alone?

Exercise 4. Profit from Poultry

1. In an experiment conducted in Ohio, out of 18 flocks of hens, the highest annual profit per 100 hens was $247. What was the average annual profit per hen in this flock?

2. The lowest profit per 100 hens was $15. What was the average annual profit per hen in this flock?

3. The average profit per 100 hens for the 18 flocks was $87. What was the average annual profit per hen in all of the flocks?

4. One hundred hens are worth about $100, the value of a good cow. The average profit from a cow is about $33 per year. How much more is realized from the 100 hens per year on an average than from a cow?

5. In an experiment in Ontario from September 1 to May 1, 25 hens which were fed buttermilk with their food yielded a profit of $11 in 8 months. 25 hens, fed beef scraps with their other food, yielded a profit of $10 in 8 months. 25 hens with no buttermilk or beef scrap gave a loss of $3. What were the profits due to the buttermilk? To the beef scrap?

Exercise 5. Home Canning Pays

1. One-tenth of an acre of tomatoes produces $\frac{3}{5}$ ton of tomatoes as an average yield. What is the income from $\frac{1}{10}$ acre of tomatoes at $9 per ton? What is the income from an acre at the same rate?

2. Three-fifths of a ton of tomatoes will average 40 dozen quarts when they are canned. The jobbers usually sell the canned tomatoes at 90 cents per dozen quarts. What do the jobbers receive for 40 dozen quarts?

3. If these cans are retailed at 15 cents a can, what are the total receipts from the 40 dozen quarts?

4. The average cost of home-canned tomatoes is 4 cents a quart. What is the average price of tomatoes at your grocery store? What is the saving per can if the tomatoes are canned at home?

5. The profits of home canning may be considered as the wages for the labor required to can the vegetables. Suppose a girl cans 100 quarts per day. What wages can she make at the profit per can which you found in Problem 4?

6. Suppose a family of five uses a can of fruit and 2 cans of vegetables on an average for every day in the year, what will be the yearly saving from home canning at 10 cents a can?

7. When tomatoes sold at 60 cents a bushel, Ellen canned $4\frac{1}{2}$ bushels of tomatoes, paying $2\frac{1}{2}$ cents each for quart cans. From each bushel she got 18 quarts which she sold at 15 cents each. What profit did she make by canning?

8. Margaret cleared $6.25 on one square rod of beans. 28 per cent of this amount she received from her canned goods and the remainder for beans sold at 5 cents a pound. How many pounds of beans did she sell?

Local data on canning should be secured by the pupils.

Exercise 6. Silage

Silage is made by chopping up corn or some other green feed and storing it in an air-tight bin called a silo. A green succulent food is thus preserved for the stock in winter. A silo furnishes food for stock in the winter in a similar way that a cold storage plant or a canning factory provides vegetables and fruits for us.

1. A dairyman put 111 tons of silage in his silo at an average cost of $2.45 per ton. What was the total cost of filling the silo?

2. How long will this silage last a herd of 25 cows, allowing 40 pounds of silage per day for each cow?

3. In a Minnesota experiment the net profit from 216 dairy cows fed on silage was $7137.59. Find the average net profit for each cow fed on silage.

4. In the same experiment the net profit from 239 cows which were not fed on silage was $5491.95. Find the average net profit for each cow not fed on silage.

5. How much more profit was made per cow on the silage-fed herd? At that rate, how much more profit would be secured from a herd of 10 cows by feeding them silage?

6. Corn yielding 75 bushels of corn to the acre will yield about 20 tons of silage per acre. If corn is worth 75 cents a bushel and silage is worth $5.00 per ton, which will be of greater value, the corn, counting the fodder worth $5 an acre, or the silage from an acre of this corn?

7. An acre of corn fed in the shape of silage will produce about 1500 pounds more milk than if fed in the shape of dry fodder and corn. How much is the extra profit in the milk secured by feeding silage if the milk sells at $2.00 per hundred pounds?

Exercise 7. Raising Calves

Courtesy International Harvester Co.

In order to encourage the raising of live stock, banks are organizing calf clubs. A state bank bought 52 calves and distributed them at cost to the children in that community. The calves were numbered and the children drew lots for them.

1. As a special inducement, the first 6 calves were sold for $25 each and the rest of the carload of 52 were sold at $32.60 each. What was the total sum paid for these calves?

2. The children were given the privilege of paying for their calves or the bank loaned them the money on 7 months' time at 5 per cent interest. Find the interest on a calf which was drawn in the $25 lot.

3. Find the interest on a calf costing $32.60 for the 7 months. What amount did a boy have to pay the bank at the end of that time if they loaned him the entire amount to pay for his $32.60 calf?

4. The average expenses for corn, oats and hay before time to put the calves on pasture were about $2 per month. The club members secured pasture at 75 cents a month. Find the total expenses for keeping a calf for 7 months if it was fed dry feed for 2 months and pastured 5 months?

5. At the end of the 7 months the calves were sold at a special sale at an average of $62.50 apiece. What was the average profit on each calf, counting out expenses and interest?

6. The boy who received the first prize of $10 for having the best cared-for calf sold his calf for $72.75. His expenses were: Cost of calf, $32.60; feed and pasture, $17.70, and interest on his note for $32.60 for 7 months at 5 per cent. What was his net profit, counting the prize?

School Achievements

In many schools pupils are now being given school credit for work done at home. Among the many different projects for which they are being given credit are: field and garden; business; cooking; sewing; poultry; music, and cow testing. The pupils are required to keep an account of their receipts and expenses and make a report on their project. From these reports they prepare and solve practical problems. If you have school projects in your school you will be able to make other good problems to supplement the lists in this book, which have been prepared from reports from pupils who have been engaged in these projects.

Exercise 8. Garden Projects

1. Mildred rented three plots of ground from her father. They measured 36 feet by 27 feet; 34 feet by 52 feet, and 12 feet by 12 feet. How many square rods did her three plots contain?

2. Mildred sold her vegetables to private families and cleared $74.75 off the three plots. How much did she make per square rod? How much would she have made at the same rate off $\frac{1}{4}$ of an acre?

3. Edna made the following sales from her garden: tomatoes, $20.84; beans, $5.10; cabbage, $4.00, and peas, $21.89. Her expenses were $4.20. What were her net profits?

4. Edward rented a plot of ground 99 feet wide by 230 feet long. What part of an acre was his plot?

5. He raised cauliflower for the first crop, receiving $131.70 for that, and squashes for the second crop, which he sold for $85.75. His expenses were $25.87. What was his net profit from the plot of ground? At that rate what would have been his profit from an acre?

6. James raised onions on a plot of 20 square rods. He sold them for $34.10, and his expenses were $6.75. Find his profit.

7. What was James' profit per square rod? At that rate what would he have made from an acre of onions?

8. Lena cleared $58.05 from her garden project. She loaned this money to her father at 3 per cent interest. What will be her interest on it at the end of one year?

9. John made $14.76 from onions on a plot $16\frac{1}{2}$ feet by 66 feet. What was his profit per square rod? What would have been his profit on an acre at that rate?

10. Richard sold 19 sacks of carrots at $5.00 per ton and received $5.30 for them. How many tons did he sell?

11. Theodore had a garden plot that measured 33 feet by 132 feet. He raised beans on it and sold them in the fall for $110.75. What was his profit per square rod?

12. What would have been his profit on an acre at the same rate?

13. In 1916 James' profit from his $\frac{1}{4}$-acre corn project was $27.00. This was $\frac{4}{5}$ more than his profit on the same field in 1915. What was his profit in 1915?

14. Harry's field is 10 rods by 16 rods. Find the cost of a three-wire fence around this field if an 80-rod spool costs $3.00.

These lists of problems give an excellent general review in addition, subtraction, multiplication, division, common and decimal fractions, measurements and percentage. They are practical because they are based upon the real data submitted by the pupils.

Exercise 9. Poultry Projects

1. Ida made a profit of \$7.87 on a flock of 15 hens. She placed this money in a savings bank at 3 per cent interest. How much would she have to her credit at the end of 6 months?

Remember that in a savings bank the cents are disregarded in computing the interest, only the whole dollars being used. The interest on Ida's account, then, will be computed on \$7 for the 6 months, and this interest added to the original deposit, \$7.87.

2. Victoria sold \$7.15 worth of poultry. Her expenses were \$2.00. What was her profit? The expenses were what part of the sales? The expenses were what part of the profit?

3. Hazel had $3\frac{1}{2}$ dozen hens. The average yield per hen for the year was 130 eggs. How many eggs did Hazel get during the year? What were these eggs worth at 25 cents a dozen? What were the eggs of one hen worth at that price?

4. Albert received \$110.80 from his poultry project. Feed and other expenses amounted to \$27.24. Find his net profit.

5. Albert set 260 eggs and got 225 chicks. What per cent of his eggs hatched?

6. He sold 134 of these chicks for \$58.77. Find the average price per chick.

7. In April Albert's hens produced 699 eggs; May, 544 eggs; June, 522 eggs; July, 429 eggs; August, 280 eggs, and September, 112 eggs. Find the total number of eggs produced during those six months. How many dozen is that? He received an average of 25 cents a dozen. How much did he receive from the sale of eggs?

8. William started his poultry project with 9 fowls worth \$12.00 His egg receipts for the year were \$18.00. His receipts from chicken sales were \$8.00 and his expenses \$7.75. William's profit was what per cent of his cash outlay?[1]

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[1] Count both the investment in fowls and expenses in the cash outlay.

Exercise 10. Business Projects

1. Herbert conducted an ice business during the summer. From May 23 to July 15, when Herbert sold cakes of ice weighing 130 pounds for 25 cents each, his receipts were $23.75. From July 15 to August 28, the price was 30 cents for 130 pounds and his receipts amounted to $56.25. How many tons of ice did Herbert sell during the summer?

2. What were Herbert's total receipts during the summer? Of this amount, he received $\frac{3}{16}$ for keeping his father's ice box filled. What was the amount of his father's ice bill? What did he receive from other sources?

3. Herbert's profit from his project equaled 60 per cent of his receipts. What were Herbert's expenses?

4. Charles paid $6 for a paper route in Tinley Park and 50 cents for a sack for his papers. His paper bills for the year 1916 were $341.50. His profit amounted to 41 per cent of his paper sales. Find the amount of his profit.

5. Anita's father paid her 3 cents on every dollar's worth of dry goods which she sold in his store. During July she made $6 and during August $12. How much did her sales amount to for the two months?

6. Besides this Anita made $1.25 filling out checks. What was the total amount of her earnings?

7. William tested 150 ears of seed corn for a farmer. The score card showed that 120 ears were good enough to plant. What per cent of the ears would do to plant?

8. The first 10 weeks of a 40-week school year Edward earned $1 a week taking the teacher to and from school and during the next 10 weeks $1.50 a week. During the rest of the term she boarded near the school, but paid Edward 25 cents each Friday during the remainder of the term for taking her to the train. How much did he earn in this way altogether?

9. Maxwell's teacher hired him to take care of her horse from January 1 to July 1, paying him $3 a month. From this and other sources he saved $21. What part of this amount did he earn by caring for the horse?

10. Catherine received 10 cents a week for delivering milk from August 6 to December 24. She spent 66 cents for Christmas presents and saved the remainder. How much did she save?

11. Lillian paid 55 cents for her reader; 50 cents for her arithmetic; 20 cents for her speller, and 35 cents for her language book. She paid 10 cents for a yard of lining to cover the books. What was the actual cost of Lillian's books for the year if at the end of the term she sold them for 40 per cent of the total cost, including the covers?

12. The Oak Lawn girls held a bakery sale to buy playground equipment. They sold: 14 cakes at 50 cents each; 15 pies at 20 cents; 6 dozen rolls at 12 cents a dozen; 12 dozen rolls at 18 cents a dozen; 8 dozen doughnuts at 18 cents a dozen; 30 loaves of bread at 10 cents a loaf, and 12 pounds of candy at 30 cents each. They found that the receipts of this sale amounted to $\frac{2}{3}$ more than the cost of the materials used. What was the cost of the materials? What was their profit?

13. Robert bought a load of vegetables from members of his garden club and sold them in the city as follows: 30 bunches of radishes for 85 cents; 26 heads of lettuce for 75 cents; 20 bunches of spinach for 50 cents; 40 quarts of beans for $2.00; 20 bunches of kohlrabi for 60 cents; 60 peppers for $1.10; 3 boxes of tomatoes for $1.50; 30 bunches of beets for 95 cents; and 24 bunches of carrots for 65 cents. His load cost him $6.80. What per cent did he gain?

14. Bessie's garden is 33 feet by 40 feet. What will be the cost of palings to enclose this garden, if the palings are 2 inches wide, are placed 2 inches apart, and cost $2.75 per hundred?

Exercise 11. Sewing Projects

1. Each of the 8 girls in a sewing class made a middy blouse. A ten-yard bolt of material, costing 11 cents a yard, made 4 blouses, and a 5-cent spool of thread was enough to stitch two blouses. What was the total cost of the 8 blouses?

2. What was the cost of each blouse? What was the saving to each member of the class when garments of this quality sold in the store for $1?

3. Genevieve bought during the year 6 pairs of stockings at 3 pairs for $1 and 10 cents worth of darning cotton for mending them. Margaret considered mending a waste of time, so each week she purchased a 10-cent pair of stockings. How much did each girl spend for stockings during the year?

4. Mary's skirt pattern called for $3\frac{1}{2}$ yards of material with nap. She found that by using goods without nap she could save $\frac{1}{7}$ of this amount. How many yards did she need of material without nap? If each kind cost $2 a yard, how much did she save by using the material without nap?

5. The material for a crocheted yoke cost Emma 60 cents. She sold the yoke for $2\frac{1}{2}$ times the cost of the materials. What did she gain?

6. Lucy priced a silk shirt and found that it would cost her $18. She bought $2\frac{3}{4}$ yards of silk, equal in quality to that of the store garment, for which she paid $3 a yard. She also used $\frac{2}{3}$ of a yard of belting at 30 cents a yard; 1 card of hook and eyes at 5 cents, and 1 spool of silk thread at 10 cents. How much did her skirt cost?

7. How much did Lucy save by making the garment herself? If it took her 10 hours to make the skirt, how much did she make per hour?

The girls of the class should be required to supplement this list with problems from their own experience.

8. Eva's little sister was constantly losing her handkerchiefs. Eva purchased 3 yards of 36-inch batiste at 12 cents a yard and 1 spool of thread for 5 cents. She cut the goods into 12-inch squares and hemmed each one neatly. How many dozen handkerchiefs did she make?

9. What was the total cost? What was the cost of each handkerchief?

10. What was saved on all the handkerchiefs if the cost at the dry-goods store was 5 cents each?

11. Florence's mother needed two 21-inch window shades 7 feet long, which would cost 55 cents each. Florence bought shade cloth 42 inches wide at 31 cents a yard; cut the goods lengthwise; allowed 3 inches for the hem at the bottom and used the old roller. What did each shade cost? What did she save on the two shades?

12. Mary's mother used six 98-pound sacks of flour last year. From each of the empty sacks she made two neat towels each 36 inches long. How much did she add to the household savings if cotton toweling sells for 10 cents a yard?

13. From 12 yards of calico, costing 10 cents a yard, Marie made 4 slip-over aprons. For each belt she used 27 inches of elastic costing 4 cents a yard. She used 6 buttons on each apron at 5 cents a dozen. Her thread cost 10 cents. The department stores charge 79 cents for these aprons. What did Marie save by making the garments at home?

Exercise 12. Cow Testing Projects

1. One of the cows tested by Alfred gave an average of 28 pounds of milk a day during March. The milk tested 4 per cent of butter fat. What was the value of the butter produced from that cow for the month, the market price being 32 cents a pound and the butter exceeding the butter fat by one-sixth?

2. The cow mentioned in Problem 1 each day consumed 15 pounds of silage, 10 pounds of alfalfa hay and 8 pounds of ground grain. Figuring silage at $4 a ton, alfalfa at $15 a ton, and the grain at $1\frac{1}{4}$ cents a pound, what did the feed cost for maintaining this cow for that month? What profit or loss resulted from keeping this cow?

3. In the month of February, 1917, one of the cows registered in a cow testing project produced on an average 25 pounds of milk a day. This tested 3.8 per cent of butter fat. At 40 cents a pound, what was the value of the butter per week if the weight of the butter exceeded the weight of the butter fat by 15 per cent?

4. What would this milk sell for if it were shipped to the city and sold at $1.80 per hundred pounds?

Exercise 13. A Cooking Club

1. A cooking club prepared a luncheon for four guests. The following articles were purchased at the grocery store for this luncheon: 1 pound of lamb chops at 25 cents; 2 cans of tomato soup at 10 cents each; 1 package of crackers at 10 cents; $\frac{1}{2}$ dozen bananas at 20 cents a dozen; 10 cents worth of nuts; 1 quart of milk at 8 cents, and 1 cake at 10 cents. Make out the grocer's bill for these supplies.

2. The member at whose house the luncheon was served donated the following articles: $\frac{1}{2}$ pint cream at 40 cents a quart; $\frac{1}{4}$ pound butter at 40 cents a pound; $\frac{1}{2}$ peck apples at $1 a bushel; 1 loaf bread at 10 cents and other materials amounting to 25 cents. Find the cost of the materials donated.

3. Find the total cost of the luncheon. What per cent of the total cost of the luncheon was the cost of the materials donated?

4. Find the cost per person for serving this meal.

These tables of reference data are designed to furnish supplementary material for problems. For example: What is the total area of the 48 states? What is the area of three Pacific Coast states? What is the average height of a story in each of the high buildings listed in Table II. Additional material similar to this should be used from the geographies.

I. Areas of the States in Square Miles

Alabama	51,998	Nebraska	77,520
Arizona	113,956	Nevada	110,690
Arkansas	53,335	New Hampshire	9,341
California	158,297	New Jersey	8,224
Colorado	103,948	New Mexico	122,634
Connecticut	4,965	New York	49,204
Delaware	2,370	North Carolina	52,426
Florida	58,666	North Dakota	70,837
Georgia	59,265	Ohio	41,040
Idaho	84,313	Oklahoma	70,057
Illinois	56,665	Oregon	96,699
Indiana	36,354	Pennsylvania	45,126
Iowa	56,147	Rhode Island	1,248
Kansas	82,158	South Carolina	30,989
Kentucky	40,598	South Dakota	77,615
Louisiana	48,506	Tennessee	42,022
Maine	33,040	Texas	265,896
Maryland	13,327	Utah	84,990
Massachusetts	8.266	Vermont	9,564
Michigan	57,980	Virginia	42,627
Minnesota	84,682	Washington	69,127
Mississippi	46,865	West Virginia	24,170
Missouri	69,420	Wisconsin	56,066
Montana	146,572	Wyoming	97,914

II. Heights of Notable New York Buildings

	Stories	Feet		Stories	Feet
Woolworth	51	785	Equitable	37	485
Metropolitan Life	50	700	Heidelberg	30	410
Singer	41	612	Municipal	24	560
Banker's Trust	39	539	Times	28	419

III. National Parks of the United States

	Acres		Acres
Lincoln Homestead	137	Chickamauga	6,195
Antietam	43	Crater Lake	159,360
Casa Grande Ruin	480	Glacier	981,681
General Grant	2560	Hawaii	74,935
Gettysburg	877	Lassen Volcano	82,880
Hot Springs Reservation	912	Mesa Verde	42,376
Platt	848	Mount Rainier	207,360
Rock Creek	1,606	Rocky Mountain	230,000
Shiloh	3,000	Sequoia	160,000
Sully's Hill	960	Wind Cave	10,522
Vicksburg	1,233	Yellowstone	2,142,702
Zoological	170	Yosemite	967,680

IV. United States Irrigation Statistics

State	Farms	Acres	Cost
Arizona	4,841	320,051	$17,677,966
California	39,352	2,664,104	72,580,030
Colorado	25,857	2,792,032	56,636,443
Idaho	16,439	1,430,848	40,977,688
Kansas	1,006	37,479	1,365,563
Montana	8,970	1,679,084	22,970,958
Nebraska	1,852	255,950	7,798,,310
Nevada	2,406	701,833	6,721,924
New Mexico	12,795	461,718	9,154,897
North Dakota	69	10,248	836,482
Oklahoma	137	4,388	47,200
Oregon	6,669	686,129	12,760,214
South Dakota	500	63,248	3,043,140
Texas	4,150	164,283	7,346,708
Utah	19,709	999,410	14,028,717
Washington	7,664	334,378	16,219,149
Wyoming	6,297	1,133,302	17,700,980

See page 248, Advanced Arithmetic, for problems on irrigation.

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For Weighing:

Scales

7 days..........................=1 week

365 days..........................=1 common year......(yr.)

366 days..........................=1 leap year

*All schools are not fully equipped with the various measures and instruments required in the study of Denominate Numbers. For this reason suitable illustrations accompany these tables, but necessarily reduced in size.

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TABLES OF DENOMINATE NUMBERS

Foot Rule, reduced to one third of its true length.

For Measuring Lengths:

Tape Measure

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Linear Measure

12 inches (in.)........ =1 foot........ (ft.)
3 feet........ =1 yard........ (yd.)
5½ yards, or 16½ feet.. =1 rod........ (rd.)
40 rods........ =1 furlong........ (fur.)
320 rods, or 5280 feet.. =1 mile........ (mi.)
1 mi. =320 rd. =1760 yd. =5280 ft. =63,360 in.

Surveyor's Chain*

Surveyors' Linear Measure

7.92 inches (in.)........ =1 link........ (l.)
25 links........ =1 rod........ (rd.)
100 links........ =1 chain........ (ch.)
80 chains........ =1 mile........ (mi.)

For Surface Measuring:

Carpenter's Square

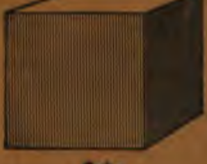
Square Measure

144 square in. (sq. in.).. =1 square foot (sq.ft.)
9 square feet (sq.ft.) =1 square yard (sq.yd.)
30¼ sq. yd., or 272¼ sq.ft =1 square rod.. (sq.rd.)
160 square rods........ =1 acre........ (A.)
640 acres........ =1 square mile (sq.mi.)
1 A. =160 sq.rd. =4840 sq.yd. =43,560 sq.ft.

Surveyors' Square Measure

16 square rods........ =1 square chain (sq.ch.)
10 square chains........ =1 acre........ (A.)
640 acres........ =1 square mile (sq.mi.)
1 square mile........ =1 section........ (sec.)
36 sections........ =1 Cong. township (T.)

Illustrating Solids:



Cube

Cubic Measure

1728 cubic inches (cu.in) =1 cubic foot.... (cu.ft.)
27 cubic feet........ =1 cubic yard.. (cu.yd.)
128 cubic feet........ =1 cord........ (cd.)
16 cubic feet........ =1 cord foot...... (cd.ft.)
8 cord feet........ =1 cord........ (cd.)

*Instead of the standard (Gunter's) chain, some surveyors use a steel tape 50 feet long, divided into foot lengths, each of these being marked off into tenths.